

# Application Costs as a Screening Instrument in Decentralized Matching

Sergei Balakin\*

Last version: Sep 15, 2020

## Abstract

We consider decentralized matching in a two-sided market of firms and workers with application costs and limited budgets. Workers choose whether they should take the risk of applying to a higher-ranked firm (with some probability of rejection) or make a safe choice. We show that application costs set by firms with uncertain capacity may be treated as a screening instrument in order to attract only strong applicants or avoid the competition. Also, we consider the case when higher-ranked firms may change their threshold of acceptance. The optimal value of this threshold is found in the case of two firms.

Key words: *decentralized matching, non-cooperative game theory, school choice, screening, exploding offers.*

JEL: *C78, D47, C72.*

## 1 Introduction

### 1.1 Overview

The majority of theoretical literature on two-sided matching markets is devoted to centralized markets. Although it is necessarily dictated by a vast number of real life problems (e.g., school allocations, the medical residency match, kidney exchange, etc.), there are many examples of markets that are not (fully) centralized (for example, the market for junior economists, paper submission to journals, dating sites, and so on). Thus, theoretical understanding of the outcomes generated by decentralized markets becomes crucial for institutional design.

---

\*The Ohio State University, balakin.1@osu.edu

In this paper, we introduce a model of decentralized matching in a two-sided market of firms and workers (institutions and applicants) with application costs and limited budgets. We show that application costs set by institutions with uncertain capacity may be treated as a screening instrument in order to avoid the competition, attract only strong workers and regulate a number of applicants. Also, the case when firms can change their threshold of acceptance is considered. We find the optimal value of this threshold in the simplest case of two firms.

We consider the matching process as a non-cooperative game where firms simultaneously choose application costs in order to maximize their profits from the entire pool of applicants. Workers in turn make a decision where to apply facing these costs and their budget constraints. Thus, in this setting we can use Nash equilibrium as a concept that allows us to successfully distinguish “good” and “bad” matchings. Due to the market structure here — and particularly because of unlimited capacity on one side, continuum of agents on the other side, and multiple applications — the canonical notion of stability cannot be fruitfully applied here.

The model is inspired (but not limited) by two markets of prospective PhD students in North American and European schools, respectively. Other applications may include transport economics (logistics, congestion, and transportation costs), online platforms (selling goods and consumer search), credit card applications and venture capital investments, dating sites and networks in general, among others. The next example shows how surprisingly different the behavior of agents may be and why these markets are interesting to solve.

## 1.2 Motivating Example

Let’s start with a simple motivating example (see Fig. 1). We have three firms  $V_2$ ,  $V_1$ , and  $V_0$  with unlimited capacity. First two of them are considered higher-ranked, so the utility of getting there for any worker  $W$  is 3. The probability that a worker will be hired is  $p$ , and it is independent among  $V_1$  and  $V_2$ . The utility of being enrolled to  $V_0$  is 1, and they are ready to accept  $W$  with probability 1. We can call  $V_0$  a safe choice then, and  $V_1$  and  $V_2$  are risky choices.

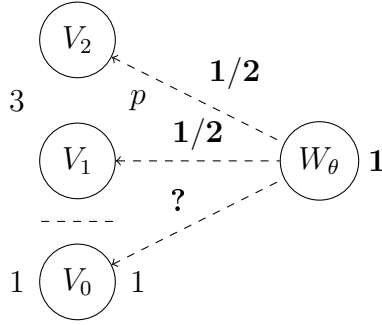


Figure 1: What application fee should be implemented by  $U_0$ ?

We will identify the worker with her type  $\theta$  and (for this example) assume that  $\theta = p$ , so  $W = W_\theta = W_p$ . Each worker knows her own type before making the decision. The safe firm values the worker with utility  $u_0(\theta) = \theta$ . Thus, the more capable the worker, the higher her chances to be hired to one of the higher-ranked firms and the more valuable for  $V_0$  she is. Assume that we have a continuum pool of uniformly distributed on  $[0, 1]$  workers, so  $\theta = p \sim U_{[0,1]}$ .

Another restriction is that the budget of each  $W$  is equal to 1, and  $V_1$  and  $V_2$  set their application costs as a half of the budget each. Thus, the question is: *what application fee must be implemented by  $V_0$ ?* Should it be less than the one set by the higher-ranked firms?

For example, if the cost for  $V_0$  is zero, a worker will have a chance to apply to all three firms. If it's higher than zero but lower than  $1/2$ , a worker faces the hard decision choosing between applying to two higher-ranked firms and diversifying her application portfolio. Finally,  $V_0$  may force a worker to choose between taking solely her safe option and applying to two higher-ranked firms by setting the application fees higher than  $1/2$ . In the first case, all workers will apply to  $V_0$  for free. In the second case, the marginal utility of making the risky choice is  $3p$ , and the marginal utility of making the safe choice is 1. Thus, a risk-neutral worker applies to two higher-ranked firms if and only if  $p \geq 1/3$ . Finally, in the third case, a worker chooses between being accepted to at least one of two higher-ranked firms (and getting  $3(1 - (1 - p)^2)$ ) and  $V_0$  (and getting 1). She would choose to apply solely to  $V_0$  if and only if  $p < 1 - \sqrt{2/3}$ . This paragraph can be summarized mathematically (here  $c_0$  is an application fee set by  $V_0$ , and  $U_0$  is an aggregate utility of  $V_0$  for the

entire pool of students  $(W_\theta)_{\theta \in [0,1]}$ :

$$\begin{aligned}
 U_0 &= \int_0^1 p(1-p)^2 dp = 0.083 && \text{if } c_0 = 0, \\
 U_0 &= \int_0^{1/3} p(1-p) dp = 0.043 && \text{if } 0 < c_0 \leq 1/2, \\
 U_0 &= \int_0^{1-\sqrt{2/3}} p dp = 0.017 && \text{if } 1/2 < c_0 \leq 1.
 \end{aligned}$$

In the first integral, we can see that everyone applies to  $V_0$ , but only the workers who get hired neither by  $V_1$  nor by  $V_2$  (probability of this event is  $(1-p)^2$ ) accept an offer from  $V_0$ . In the second integral, two thirds of the workers with higher type do not even apply to  $V_0$ . The lower half applies to  $V_0$  and either  $V_1$  or  $V_2$ . Thus, they accept an offer from  $V_0$  with probability  $1-p$ . Finally, in the third integral only the workers with type  $p < 1 - \sqrt{2/3}$  are going to apply to  $V_0$ . They all accept an offer from  $V_0$ , since it is their only application.

This example demonstrates that the safer firm needs to implement zero costs in order to attract better workers and get more profit. But what if the difference between firms is not so drastic? For example, assume now that  $a = 3/2$  and calculate the aggregate utility in this case.

$$\begin{aligned}
 U_0 &= \int_0^1 p(1-p)^2 dp = 0.083 && \text{if } c_0 = 0, \\
 U_0 &= \int_0^{2/3} p(1-p) dp = 0.123 && \text{if } 0 < c_0 \leq 1/2, \\
 U_0 &= \int_0^{1-1/\sqrt{3}} p dp = 0.089 && \text{if } 1/2 < c_0 \leq 1.
 \end{aligned}$$

We can see that, maybe against our expectations,  $V_0$  now should implement the same fees as the higher-ranked firms forcing applicants to make the unpleasant decision and taking a substantial part of the pool away from  $V_1$  and  $V_2$ . Moreover, if we continue to decrease  $a$ , we will realize that starting from  $a = 1.26$  the most profitable strategy for  $V_0$  will be setting the highest possible costs equal to 1. We will call this strategy Exploding Offer. Indeed, the lower-ranked firm makes a job offer to a worker right now. However, should she reject it, she would be able to apply to all the higher-ranked firms, but there will be no safe option anymore.

We can observe the similar picture analyzing the quality of workers. The more high-skilled workers are on the market, the less incentives the safer firm has to set higher costs, and vice versa.

The most natural (and inspiring) example of this model is the market of prospective PhD students. Here firms are universities, and workers are PhD applicants. More precisely, we are interested in its difference between North American and European schools. Namely, European PhD admission is mostly characterized by zero application costs, which is absolutely not the case in the U.S. universities. For our model, we find conditions for equilibria when costs are negligible (European equilibrium), costs are a substantial part of the budget (American equilibrium), costs imply exploding offers (the worse university tries to “steal” the better student), in addition to other equilibria. In particular, we show that exploding offers are beneficial for all schools and not only for weaker universities that implement them.

Another noticeable setting is a model of opening firms (stores) inside the city limits. Here the budget of a customer is time that she is willing to spend on shopping. Depending on customer needs (or willingness to spend money)  $\theta$ , she might choose a store with worse quality but better chances to find the good she needs, and vice versa. In the example above, this low-quality store should choose whether it wants to open right in the city center (and provide zero transportation costs for the citizen) or somewhere on the outskirts.

Publications in scientific journals is another great example that satisfies the model. Depending on quality of the paper  $\theta$ , an author may send the paper to a mediocre but safe journal or try to publish higher with some probability of reject. We can assume that the market here is decentralized and check if there is an equilibrium which describes exactly what is happening on this market in real life. Our findings tell us that the equilibrium that forces authors to send their manuscripts only to one journal at the time ( $c_i = 1$  for all  $i$ ) is approachable when the distribution of applicants is skewed towards low types (the majority of papers are relatively bad).

Also, with some adjustments, our model may be applied to online platforms (selling goods and consumer search), credit card applications in the U.S., venture capital investments, and online dating sites, among others.

We continue with the general description of the model in Section 2 and consider the case of two universities in Section 3. For them, we will describe all the equilibria and find an optimal threshold for a higher-ranked firm. Section 4 is devoted to a richer case of three firms when  $p = \theta$  with substantial examples provided. Section 5 contains some general results. Proofs and complex calculations are taken out to Appendix.

## 2 The Model

Consider a two sided-market with  $m + 1$  participants on one side and a continuum of participants on another side. These participants form set  $V = \{V_1, V_2, \dots, V_m, V_0\}$  of firms (institutions) and set  $W = \{W_\theta\}_{\theta \in [0,1]}$  of workers (applicants), respectively. All applicants have an equal budget (endowment) that we normalize to 1. Institution  $V_i$ ,  $i \in \{1, 2, \dots, m, 0\}$  introduces application cost  $c_i \geq 0$ , and workers apply to firms paying those costs up to their budget constraints. Formally,

$$\sum_{i=1,2,\dots,m,0} c_i I\{W_\theta \text{ applies to } V_i\} \leq 1 \quad (1)$$

for any  $\theta \in [0, 1]$ . Here,  $I\{W_\theta \text{ applies to } V_i\}$  is equal to 1 if  $W_\theta$  applies to  $V_i$ , and zero otherwise.

Type  $\theta$  of each worker  $W_\theta$  is a random variable on segment  $[0, 1]$  with cumulative distribution function  $F$ . This type is entirely disclosed only to a worker herself. For any  $i$ , firms  $V_i$  observe noisy types  $x_i = \theta + \varepsilon_i$  where  $\varepsilon_i$  are mutually independent and identically distributed symmetric random variables with cdf  $G$ . We assume that  $G$  is absolutely continuous. Subindexes may be omitted if it does not result in any confusion.

A worker applying to  $V_0$  gets accepted with probability 1. A worker applying to  $V_i$  ( $i \in \{1, \dots, m\}$ ) gets accepted if her observed type  $x_i$  is higher than  $\bar{x}_i$ . These  $\bar{x}_i$  are common knowledge and may either be given exogenously or defined by firms in advance. In any case, vector  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_m)$  is disclosed to both workers (before they make their decision about where to apply) and firms (before they implement costs). Let  $p$  be the probability of being enrolled by  $V_i$  ( $i \in \{1, \dots, m\}$ ). Then

$$p = Pr\{x \geq \bar{x}\} = Pr\{\varepsilon \geq \bar{x} - \theta\} = 1 - G(\bar{x} - \theta). \quad (2)$$

All firms value workers depending on their type  $\theta$ . Let monotonic non-decreasing functions  $u_i(\theta)$  be the utilities of firms  $V_i$ ,  $i \in \{1, \dots, m, 0\}$ . (We will also assume that  $u_0(\theta) \geq 0$  for all  $\theta$ .) Then aggregate expected utility functions for firms are

$$U_i = \int_0^1 u_i(\theta) Pr\{S_\theta \text{ matches } V_i\} dF(\theta) \quad (i = 1, \dots, m, 0).$$

Here, by “ $W_\theta$  matches  $V_i$ ” we mean that  $W_\theta$  had applied to  $V_i$ , was made an offer, and accepted it.

Finally, let  $w(i)$  be a worker’s utility of matching with  $V_i$ :  $w(0) = 1$  and  $w(i) = a > 1$  if  $i \in \{1, \dots, m\}$ . Thus, we can see that firms  $V_1, \dots, V_m$  are

higher-ranked than  $V_0$ , and it is harder to get there. Lower-ranked  $V_0$  may be called a safe choice. If a worker gets hired by both higher-ranked and lower-ranked firms, she always chooses the first one. Also, we assume that if an applicant gets enrolled to  $k$  of  $m$  higher-ranked firms, she chooses one of them with equal probability  $1/k$ .

Now we are ready to describe the entire game. First, thresholds  $\bar{x}$  get determined (either by “nature” or by institutions themselves). Then firms  $V_1, V_2, \dots, V_m, V_0$  independently set their nonnegative costs  $c_1, c_2, \dots, c_m, c_0$  in order to maximize their expected utilities  $U_i$ . At the next step, workers apply according to their types and budget constraint (1) in order to maximize their own expected payoffs  $w$ . Unmatched workers and firms receive zero utility.

Let’s describe our game formally. For simplicity, consider  $\bar{x}$  as given by nature. Thus, setting a threshold is not a part of a firm’s strategy profile. We define the strategy set of the game as

$$(A_1, \dots, A_m, A_0, (B_\theta(c))_{\theta \in [0,1], c \in ([0,1] \cup \{1+\delta\})^{m+1}}).$$

Here, a strategy set for firm  $i$  is denoted by  $A_i = [0, 1] \cup \{1 + \delta\}$  ( $i \in \{1, \dots, m, 0\}$ ). A strategy set for worker  $\theta$  in each information set defined by firms’ actions  $c = (c_1, \dots, c_m, c_0) \in (A_1, \dots, A_m, A_0)$  is

$$B_\theta(c) = (I\{W_\theta \text{ applies to } V_i\})_{i=0}^m, \quad \text{such that } \sum_{i=0}^m I\{W_\theta \text{ applies to } V_i\}c_i \leq 1.$$

Again,  $I\{\cdot\}$  is an indicator here. Value  $1 + \delta$  in  $A_i$  describes the case when  $V_i$  does not want to enter the market and remains with zero payoff (we could use any  $c_i > 1$  instead).

Should firms be allowed to choose  $\bar{x}_i \in \bar{X}_i$ , sets  $\bar{X}_i \in \mathbb{R}$  ( $i \in \{1, \dots, m\}$ ) will be included in the strategy set of the game:

$$(\bar{X}_1, \dots, \bar{X}_m, (A_1(\bar{x}))_{\bar{x}}, \dots, (A_m(\bar{x}))_{\bar{x}}, (A_0(\bar{x}))_{\bar{x}}, (B_\theta(c, \bar{x}))_{\theta, c, \bar{x}}).$$

We can see that now firm and workers need to define their strategies for any possible value of  $\bar{x}$ .

Note that technically this market may be called the game only from the firms’ perspective. Workers make decisions in order to maximize their own profits, and these decisions do not depend on decisions of other applicants. Nevertheless, we include all the possible sets of workers’ actions for completeness.

Throughout the entire paper, we consider only pure strategy subgame perfect Nash equilibria. Moreover, from the whole set of Nash equilibria (NE)

that deliver the same payoffs to all the participants, we will distinguish a particular NE (and call it cNE) with maximum and symmetric application fees for firms  $V_1, \dots, V_m, V_0$ . Thus, we implicitly assume a sort of lexicographic preferences for firms: first finding sets of Nash equilibria using payoffs that do not value costs; then finding contraction of NE in each set by maximizing costs for the firms. The next definition holds.

**Definition.** Let  $C$  be a set of pure strategy Nash equilibria for  $V_1, V_2, \dots, V_m, V_0$  where for each element of this set (i.e., for each strategy profile  $c \in C$ ) utilities of all the universities do not change. Let  $\mathcal{C}$  be a collection of all these sets. Precisely,

$$\mathcal{C} = \{C \in (A_1, \dots, A_m, A_0) : \forall i \in \overline{0, m} \forall c, c' \in C (c \text{ is NE}, u_i(c) = u_i(c'))\}.$$

Define cNE as an injective function  $\text{cNE} : \mathcal{C} \rightarrow (A_1, \dots, A_m, A_0)$ , such that  $\text{cNE}(C) = c^*$  if and only if the next statements hold:

1.  $c^* = (c_1^*, \dots, c_1^*, c_0^*) \in C$ ,
2. if  $\exists c = (c_1, \dots, c_1, c_1) \in C$  then

$$c_1^* = c_0^* = \max_{c=(c_1, \dots, c_1) \in C} c_1,$$

otherwise

$$c_1^* = \max_{c=(c_1, \dots, c_1, c_0) \in C} c_1, \quad c_0^* = \max_{c=(c_1^*, \dots, c_1^*, c_0) \in C} c_0.$$

Note that this definition is correct in the sense that corresponding  $c$  exists and unique for any  $C$ .

If there is no confusion, we may omit subindices in utility functions for higher-ranked firms due to symmetry: for any  $i = 1, \dots, m$ ,  $u(p) = u_i(p)$ ,  $U = U_i$ .

Before considering different cases, let's return to our equation (2) that defines probability  $p$  and formulate important lemma.

**Lemma.** Probability  $p$  of being hired by a higher-ranked firm is equal to type  $\theta$  of a worker if and only if the threshold is equal to  $1/2$  and the noise is uniformly distributed on segment  $[-1/2, 1/2]$ :  $\bar{x} = 1/2$ ,  $\varepsilon \sim U_{[-1/2, 1/2]}$ .

**Proof.** Let  $p = \theta$ . Then from (2) we have  $G(\bar{x} - \theta) = 1 - \theta$  for any  $\theta \in [0, 1]$ . Since noise  $\varepsilon$  is symmetric, we have  $G(x) = 1 - G(-x)$  for any  $x$ . Combining last two equations, we obtain

$$G(\theta - \bar{x}) = 1 - G(\bar{x} - \theta) = 1 - (1 - \theta) = \theta.$$



For  $\theta = 1/2$ , we have  $G(1/2 - \bar{x}) = 1/2$  and  $G(\bar{x} - 1/2) = 1/2$ . Since  $G$  is absolutely continuous and, therefore, strictly increasing, we necessarily have  $1/2 - \bar{x} = \bar{x} - 1/2$  which implies  $\bar{x} = 1/2$ . Thus,  $G(1/2 - \theta) = 1 - \theta$ . Considering  $1/2 - \theta = t$ , we get  $G(t) = 1/2 + t$  which is exactly the cdf of  $U_{[-1/2, 1/2]}$ .

The opposite direction of proof is obvious. ■

Thus, we can conclude that the type of a worker may be equal to the probability of being accepted only in one possible scenario. We will consider this particular case later for  $m \geq 2$ . Meanwhile, we will focus on the simplest case  $m = 1$  and examine behavior of agents under different values of parameters.

### 3 Case of two firms: $m = 1$

We start with the simplest case of only two institutions: higher-ranked but risky firm vs. lower-ranked but safe firm.

#### 3.1 General case and Equilibria

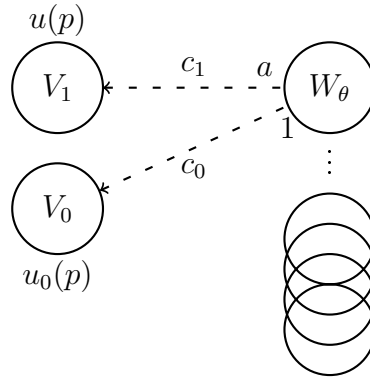


Figure 2: Case of two firms.

The game is drawn on Fig 2. The main concern of a worker under given costs  $c$  is whether she should choose a higher-ranked firm (choice with uncertainty) or a lower-ranked one (safe choice). The marginal utility of applying to  $V_1$  is  $ap = a(1 - G(\bar{x} - \theta))$ . The marginal utility of applying to  $V_0$  is 1. Therefore, should a worker of type  $\theta$  choose between the risky and the safe choice, she chooses a higher-ranked firm if and only if  $p > 1/a$ . Let's define the pivotal value of  $\theta$  by  $\hat{\theta}$ . Thus,  $\hat{\theta}$  is the root of equation

$$G(\bar{x} - \hat{\theta}) = 1 - \frac{1}{a}.$$

Since  $G$  is strictly monotone and for any  $a > 1$  we have  $0 < 1 - 1/a < 1$ ,  $\hat{\theta}$  is always unique (if it exists).

The next proposition describes all the possible equilibria in this game.

**Proposition 1.** Let

$$\begin{aligned} U_0^1 &= \int_0^1 u_0(\theta)G(\bar{x} - \theta)dF(\theta), & U_0^2 &= \int_0^{\hat{\theta}} u_0(\theta)dF(\theta), \\ U_1^1 &= \int_0^1 u_1(\theta)(1 - G(\bar{x} - \theta))dF(\theta), & U_1^2 &= \int_{\hat{\theta}}^1 u_1(\theta)dF(\theta). \end{aligned}$$

Then the game on Fig. 2 has the following set of (subgame perfect) Nash equilibria ( $0 \leq c_1 \leq 1$ ,  $0 \leq c_0 \leq 1$ ):

1. if  $U_0^1 > U_0^2$ ,  $U_1^1 > U_1^2$ ,  $U_1^1 \geq 0$ , then the NE set is  $(c_1, c_0)$ ,  $c_1 + c_0 \leq 1$  with cNE  $(1/2, 1/2)$
2. if  $U_0^1 > U_0^2$ ,  $U_1^1 < U_1^2$ ,  $U_1^1 \geq 0$ , then the NE set is  $(c_1, 0)$  with cNE  $(1, 0)$
3. if  $U_0^1 > U_0^2$ ,  $U_1^1 < 0$ ,  $U_1^2 \geq 0$ , then the only NE is  $(1 + \delta, 0)$
4. if  $U_0^1 < U_0^2$ ,  $U_1^1 < U_1^2$ ,  $U_1^2 \geq 0$ , then the NE set is  $(c_1, c_0)$ ,  $c_1 + c_0 > 1$  with cNE  $(1, 1)$
5. if  $U_0^1 < U_0^2$ ,  $U_1^1 > U_1^2$ ,  $U_1^1 \geq 0$ , then the NE set is  $(0, c_0)$  with cNE  $(0, 1)$
6. if  $U_1^1 < 0$ ,  $U_1^2 < 0$ , then the NE set is  $(1 + \delta, c_1)$  with cNE  $(1 + \delta, 1)$ .

The proof is in Appendix. ■

The degenerate case 6 is probably the most obvious one.  $V_1$  does not enter the market, and  $V_0$  gets all the workers. Case 3 is a little bit different.  $V_1$  would like to enter if it could take only workers with types above the value  $\hat{\theta}$ . However,  $V_0$  is not interested in splitting the market this way and sets zero costs. Under these conditions (when workers can apply to both firms), it is not profitable for  $V_1$  to enter.

Cases 2 and 5 are symmetric in the sense that one firm would like to split the market by setting high costs, so workers could not apply to both institutions. Thus, the only option for the other firm to fight for the entire pool of workers is to set zero costs.

Cases 1 and 4 are the most interesting ones. In case 1, both firms are interested in sharing the market.  $V_1$  finds more profitable to have applications from the entire pool of workers but not only from the top, and  $V_0$  prefers to take applicants who get rejected by  $V_1$  rather than to deal only with bad workers. In the next chapter, we will call it European Equilibrium for higher

$m$ . In case 4, both firms would like to have their own pool of workers:  $V_1$  would like to get rid of bad workers' applications, and  $V_0$  does not want to accept only workers who were rejected by  $V_1$ . This is American Equilibrium and Exploding Offer together (they will be separated for  $m \geq 2$ ).

Before moving to examples, we will answer one important question. Assuming that the higher-ranked university can choose  $\bar{x}$  itself, what is going to be the optimal value of  $\bar{x}$  for  $V_1$ ?

### 3.2 Optimal $\bar{x}$ for Linear Utility Functions

Consider  $u_0(\theta) = \theta$  and  $u_1(\theta) = \theta - \alpha$  ( $\alpha \geq 0$ ). It means that the lower-ranked firm would be happy to hire any worker from the pool, but the higher-ranked firm gets profit only from workers with type  $\theta > \alpha$ . But does it mean that  $V_1$  should set the threshold  $\bar{x}$  equal to  $\alpha$  in the case of uniformly distributed type  $\theta$ ?

Let the noise  $\varepsilon$  be distributed uniformly on segment  $[-l, l]$ ,  $l \leq 1/2$ :  $\varepsilon \sim U_{[-1/2, 1/2]}$ . Then

$$p = Pr\{x \geq \bar{x}\} = 1 - G(\bar{x} - \theta) = \begin{cases} 0, & \theta < \bar{x} - l, \\ \frac{1}{2} - \frac{\bar{x} - \theta}{2l}, & \bar{x} - l \leq \theta \leq \bar{x} + l, \\ 1, & \theta > \bar{x} + l. \end{cases}$$

We have three different cases to consider:

1.  $\bar{x} < l$
2.  $l \leq \bar{x} \leq 1 - l$
3.  $\bar{x} > 1 - l$

1. If  $\bar{x} < l$ , we have the only cNE  $(1, 0)$  and the optimal value of  $\bar{x}$  is  $2\alpha - l$ . This happens due to the bounded support of  $\theta$  and a firm's updating. Indeed, let's assume a firm gets a signal that a type of a worker is  $x = l/2$ . Then the initial type  $\theta$  must be uniformly distributed on the segment  $[0, 3l/4]$  (there are no values of  $\theta$  below zero). Taking it into account and calculating the conditional mean, a firm obtains  $\mathbb{E}(\theta | x = l/2) = 3l/8$ . Therefore, if  $\alpha = 3l/8$ , then the optimal threshold should be  $\bar{x} = l/2$ . Setting a threshold below  $l/2$ , a firm hires workers that would deliver negative utility in expectation. Setting a threshold above  $l/2$ , a firm loses some workers that would deliver expected profit. This optimal value exists only if  $2\alpha - l < l$  which means  $\alpha < l$ . If  $\alpha$  is small enough, the optimal  $\bar{x} = 2\alpha - l$  will be less than  $\alpha$ .

2. On this segment, updating does not imply any shifts since the bounds cannot be reached. The only cNE here is still  $(1, 0)$ , and the optimal value of  $\bar{x}$  is  $\bar{x} = \alpha$ . That happens when  $l \leq \alpha \leq 1 - l$ . If the range of the noise is infinitesimally small, we have  $\bar{x} = \alpha$  almost everywhere.

3. On this segment, we have an equilibrium change from  $(1, 0)$  to  $(1, 1)$  and then to  $(0, 1)$ . After the first change, the behavior changes completely.

Table 1 summarizes the findings for  $l = 1/2$  and  $a = 2$ . Note that for  $l = 1/2$  the only point where  $\bar{x} = \alpha$  is  $1/2$ .

$\alpha$	$[0, 1/2)$	$[1/2, 0.64)$	$[0.64, 0.81)$	$[0.81, 1]$
$\bar{x}$	$2\alpha - 1/2$	$2\alpha - 1/2$	0.78	$\frac{1}{2} + \alpha - \frac{1}{2}\sqrt{5 - 8\alpha + 4\alpha^2}$
$U_1$	$\frac{1}{2} - \alpha + \frac{2}{3}\alpha^3$	$\frac{2}{3}(1 - \alpha)^3$	$0.12 - 0.14\alpha$	...

Table 1: Optimal  $\bar{x}$  for different  $\alpha$  if  $a = 2, F(\theta) = \theta, G(x) = 2x + 1$

## 4 Case of three firms: $m = 2$

For simplicity, we consider only the most interesting case  $p = \theta$  (and, therefore,  $\bar{x} = 1/2, \varepsilon \sim U_{[-1/2, 1/2]}$ ) throughout the next two chapters.

This case includes competition among higher-ranked firms that changes the picture substantially (see Fig. 3). Also, under some values of costs workers may choose not only between the higher-ranked ( $\theta \geq 1/2$ ) and the lower-ranked ( $\theta < 1/2$ ) universities, but between applying to either both risky firms and the safe one (for example,  $c_1 = c_2 = 1/2, c_0 = 1$ ). In this case,  $\hat{\theta}$  is different. The probability of being accepted to at least one of the good universities is  $(1 - (1 - p)^2)$ , so each student chooses between getting  $a(1 - (1 - p)^2)$  and 1. Solving this, we find that if  $p \geq 1 - \sqrt{1 - 1/a}$ , a student applies to  $V_1$  and  $V_2$ , and otherwise she applies to  $V_0$ .

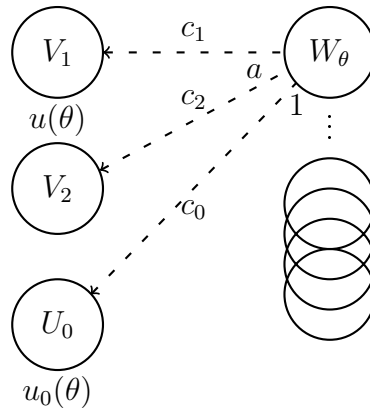


Figure 3: Case of three firms.

Proposition 2 in Appendix describes all the expected utilities of the firms for different actions. For each given  $a$ ,  $F(\theta)$ ,  $u(\theta)$ , and  $u_0(\theta)$  we can find all best responses and thus all the possible pure strategy equilibria in the game. Consider several cases.

## 4.1 Positive Valuation of Workers by All Firms

Assume first that both higher-ranked firms welcome all types of workers.

**Example 4.**  $a \geq 2$ ,  $F(\theta) = \theta$ ,  $u(\theta) = \theta$ ,  $u_0(\theta) = \theta$ . The only set  $C_{EE}$  of pure strategy Nash equilibria here satisfies the following:

$$c_1 + c_2 + c_0 \leq 1.$$

Firms' expected payoffs are  $U = 0.2083$ ,  $U_0 = 0.0833$ . From the entire pool of workers, 33% go to  $V_0$  in expectation and other 67% go to either  $V_1$  or  $V_2$ . Each firm gets exactly 1/3 of the entire pool.

The cNE here is  $(1/3, 1/3, 1/3)$ . Thus, the costs form a negligible portion of the budget and go to zero when we increase number  $m$  of good firms. We call this *European Equilibrium*.

There exists only degenerate cNE  $(0, 0, 1)$  if  $a < 2$ . Also, we can observe European Equilibrium under  $a \geq 1.8002$  if the pool is skewed towards high-quality workers ( $F(\theta) = \theta^2$ ) and under  $a \geq 2.2497$  if the pool contains a lot of low-quality workers ( $F(\theta) = 2\theta - \theta^2$ ). See the resulting Table 2.

$F(\theta)$	$(1/3, 1/3, 1/3)$	$(0, 0, 1)$
$\theta^2$	$a \geq 1.8002$	$a \leq 1.8002$
$\theta$	$a \geq 2$	$a \leq 2$
$2\theta - \theta^2$	$a \geq 2.2497$	$a \leq 2.2497$

Table 2: Equilibria with Positive Valuation for different  $F$

## 4.2 Picky Firms

Now assume that the higher-ranked firms are not interested in bad workers.

**Example 5.**  $a = 2$ ,  $F(\theta) = \theta$ ,  $u(\theta) = \theta - 1/2$ ,  $u_0(\theta) = \theta$ . We can see that  $V_1$  and  $V_2$  would like to avoid matching with all workers whose type is  $\theta < 1/2$ . There are two sets  $C_{AE}$  and  $C_D$  of pure strategy Nash equilibria

that correspondingly satisfy the following constraints:

$$\begin{cases} c_1 + c_2 + c_0 > 1, \\ c_1 + c_2 \leq 1, \\ c_1 + c_0 \leq 1, \\ c_2 + c_0 \leq 1, \\ c_1 \geq c_0, c_2 \geq c_0, \end{cases} \quad \begin{cases} c_1 + c_2 > 1, \\ c_0 = 0. \end{cases}$$

Firms' expected payoffs are  $U = 0.0495$ ,  $U_0 = 0.0833$  for  $C_{AE}$  and  $U = 0.0417$ ,  $U_0 = 0.01667$  for  $C_D$ . From the entire pool of workers, 38% go to  $V_0$  in expectation, 58% go to either  $V_1$  or  $V_2$ , and 4% remain unmatched in the first case.

The cNE in the degenerate second case is  $(1, 1, 0)$ . For  $C_{AE}$ , the cNE is  $(1/2, 1/2, 1/2)$ . We can see that costs here form a substantial part of the workers' budgets, although they leave some room for flexibility (workers can apply to both risky and safe firms). We call it *American Equilibrium*. The difference with European Equilibrium from Example 4 here is that costs do not converge to zero when  $m \rightarrow \infty$ .

**Example 6.**  $a = 1.25$ ,  $F(\theta) = \theta$ ,  $u(\theta) = \theta - 1/2$ ,  $u_0(\theta) = \theta$ . Then  $C_{EO}$  can be defined by the following inequalities:

$$\begin{cases} c_1 + c_2 \leq 1, \\ c_1 + c_0 > 1, c_2 + c_0 > 1, \\ c_1 \leq c_0, c_2 \leq c_0. \end{cases}$$

Firms' expected payoffs are  $U = 0.0593$ ,  $U_0 = 0.1528$ . From the entire pool of workers, 55% go to  $V_0$  in expectation, 42% go to either  $V_1$  or  $V_2$ , and 3% remain unmatched.

The cNE here is  $(1/2, 1/2, 1)$ . We know this situation as *Exploding Offer*. Indeed, the choice between solo  $V_0$  and both higher-ranked firms literally means that the safe firm gives you an offer right now. You can accept it and thus lose an opportunity to be considered by better firms, or you can reject it, but in this case the offer explodes and  $V_0$  is not in the pool of available firms anymore. Such an aggressive policy allows  $V_0$  to take 55% of the market. Notice that it works only if the difference between firms is small and, what is maybe even more important, workers' relative value of being hired is much higher.

Examining the entire range of  $a$ , we can see that there exists only degenerate cNE  $(1, 1, 0)$  if  $a > 2$  and  $(0, 0, 1)$  if  $a < 1.1655$ . Two equilibria described in Example 5 may be observed for all  $\sqrt{3} \leq a \leq 2$ , and only one

American Equilibrium is allowed when  $1,2622 < a < \sqrt{3}$ . Exploding Offer is supported by  $1.1655 \leq a < 1.2622$ . Another interesting situation may be observed under  $a = 1.2622$ .

**Example 7.**  $a = 1.2622$ ,  $F(\theta) = \theta$ ,  $u(\theta) = \theta - 1/2$ ,  $u_0(\theta) = \theta$ . There are two sets  $C'_{AE}$  and  $C_{EO}$  of pure strategy Nash equilibria that correspondingly satisfy the following constraints:

$$\begin{cases} c_1 + c_2 + c_0 > 1, \\ c_1 + c_2 \leq 1, \\ c_1 + c_0 \leq 1, \\ c_2 + c_0 \leq 1, \end{cases} \quad \begin{cases} c_1 + c_2 \leq 1, \\ c_1 + c_0 > 1, \quad c_2 + c_0 > 1, \\ c_1 \leq c_0, \quad c_2 \leq c_0. \end{cases}$$

As before,  $cNE(C'_{AE}) = (1/2, 1/2, 1/2)$ ,  $cNE(C_{EO}) = (1/2, 1/2, 1)$ . Firms' expected payoffs are  $U = 0.0451$ ,  $U_0 = 0.1481$  and  $U = 0.0595$ ,  $U_0 = 0.1481$ , respectively. We can see that going down through  $a = 1.2622$ ,  $V_0$  jumps from American Equilibrium to Exploding Offer, delivering higher payoffs to everyone. Not taking into account degenerate equilibria, we can see that utility of  $V_0$  monotonically grows while  $a$  goes down from 2 to 1.1655, but utility of the higher-ranked firms goes in the opposite direction with a jump at  $a = 1.2622$ . For the full picture, see Table 3.

$a$	$(0, 0, 1)$	$(1/2, 1/2, 1)$	$(1/2, 1/2, 1/2)$	$(1, 1, 0)$
$1 < a \leq 1.1655$	+	-	-	-
$1.1655 \leq a \leq 1.2622$	+	+	-	-
$1.2622 \leq a \leq \sqrt{3}$	-	+	+	-
$\sqrt{3} \leq a \leq 2$	-	-	+	+
$2 \leq a$	-	-	-	+

Table 3: Equilibria for picky firms and different  $a$  ( $F(\theta) = \theta$ )

### 4.3 Picky Firms and Low-Quality Workers

**Example 8.**  $a = 2$ ,  $F(\theta) = 2\theta - \theta^2$ ,  $u(\theta) = \theta - 1/2$ ,  $v_0(\theta) = \theta$ . There are two different sets  $C_{AE}$  and  $C_{OA}$  of pure strategy Nash equilibria here:

$$\begin{cases} c_1 + c_2 + c_0 > 1, \\ c_1 + c_2 \leq 1, \quad c_1 + c_0 \leq 1, \quad c_2 + c_0 \leq 1, \\ c_1 \geq c_0, \quad c_2 \geq c_0 \end{cases} \quad \text{and} \quad \begin{cases} c_1 + c_2 > 1, \\ c_1 + c_0 > 1, \\ c_2 + c_0 > 1. \end{cases}$$

Firms' expected payoffs a) in the first case:  $U = 0.0036$ ,  $U_0 = 0.1146$ ; b) in the second case:  $U = 0.0156$ ,  $U_0 = 0.1667$ . Thus, the second equilibrium is

Pareto optimal for the firms. From the entire pool of workers 58% go to  $V_0$  in expectation, 39% go to either  $V_1$  or  $V_2$ , and 3% remain unmatched in the first case, and 75% go to  $U_0$  in expectation, 17% go to either  $V_1$  or  $V_2$ , and 8% remain unmatched in the second case.

We can see that  $cNE(C_{AE}) = (1/2, 1/2, 1/2)$  and  $cNE(C_{OA}) = (1, 1, 1)$ . In the second equilibrium, firms use the hardest possible screening to force low-quality workers apply only to  $V_0$ . Thus, each worker is allowed to submit only one application. This is a typical market of scientific journals and authors (if we consider it decentralized). There are good journals where a paper can easily be rejected and mediocre ones where a paper will most likely be published. Depending on the quality  $\theta$  of the article, the author must make a decision where to submit. The major cost in this case is probably time, not money, and it is more or less the same for any journal. It might have been allowed for papers to be submitted to two different journals (the first equilibrium), but this is not a Pareto optimal strategy for journals.

**Example 9.**  $a = 1.25$ ,  $F(\theta) = 2\theta - \theta^2$ ,  $u(\theta) = \theta - 1/2$ ,  $u_0(\theta) = \theta$ . There are two different sets  $C'_{OA}$  and  $C'_{EO}$  of pure strategy Nash equilibria again:

$$\left\{ \begin{array}{l} c_1 + c_0 > 1, \quad c_2 + c_0 > 1, \\ c_1 \geq c_0, \quad c_2 \geq c_0 \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} c_1 + c_2 \leq 1, \\ c_1 + c_0 > 1, \quad c_2 + c_0 > 1, \\ c_1 \leq c_0, \quad c_2 \leq c_0. \end{array} \right.$$

Firms' expected payoffs a) in the first case:  $U = 0.0064$ ,  $U_0 = 0.2987$ ; b) in the second case:  $U = 0.0188$ ,  $U_0 = 0.193$ . From the entire pool of workers 96% go to  $V_0$  in expectation, 3% go to either  $V_1$  or  $V_2$ , and 1% remain unmatched in the first case, and 80% go to  $V_0$  in expectation, 18% go to either  $V_1$  or  $V_2$ , and 2% remain unmatched in the second case.

Here,  $cNE(C'_{OA}) = (1, 1, 1)$  and  $cNE(C'_{EO}) = (1/2, 1/2, 1)$ . To choose a more appropriate equilibrium, fix  $c_0 = 1$  and focus on the behavior of  $V_1$  and  $V_2$ . Their part of the game may be characterized by the next payoffs table:

$V_1 \setminus V_2$	$1/2 : c_1 + c_2 \leq 1$	$1 : c_1 + c_2 > 1$
$1/2 : c_1 + c_2 \leq 1$	0.0188	0.0064
$1 : c_1 + c_2 > 1$	0.0064	0.0064

We can see that strategies 1 are weakly dominated by strategies 1/2 for both firms. Exploding Offer cNE  $(1/2, 1/2, 1)$  looks more reasonable here.

Note that pools with high-quality workers (for example,  $F(\theta) = \theta^2$ ) do not give us any information because no pure strategy Nash equilibria exist there. It is not the case in the next subsection though.

We can conclude that our model explains the differences in application costs on American and European Econ PhD markets by different evaluation



of students. Prestigious universities in the US are not interested in enrolling low-quality students whose possible struggling with the program and bad placement may worsen the reputation of a school and decrease their teaching standards. On the other hand, universities in Europe are less competitive and more socially oriented. More positive evaluation of students in Europe and existence of European equilibrium even with skewed to the right densities may also imply that prospective PhD students' pools in Europe are of higher quality on average. Of course, this is just a speculation, since we do not know all the mechanisms behind costs setting. For example, it may be the case that in some countries in Europe governments require implementing zero costs.

## 5 General Case of $m + 1$ Firms

Consider necessary and sufficient conditions for some of equilibria from the previous chapter in the general case.

**Proposition 3.** European Equilibrium with  $C_{EE} = \{c : c_1 + c_2 + \dots + c_m + c_0 \leq 1\}$  and  $cNE(C_{EE}) = (1/m, 1/m, \dots, 1/m, 1/m)$  exists if and only if

$$\begin{aligned} \int_0^1 u_0(p)(1-p)^m dF(p) &\geq \int_0^{1-\sqrt[t]{1-\frac{1}{a}}} u_0(p)(1-p)^{m-t} dF(p) \quad (t = 1, 2, \dots, m), \\ \int_0^{1/a} u(p)(1-p)^{m-1} p dF(p) &\geq 0, \\ \int_0^{1/a} u(p)(1-(1-p)^m) dF(p) &\geq 0. \end{aligned}$$

**Proposition 4.** American Equilibrium with  $C_{AE} = \{c : c_i + c_j + c_0 > 1, c_i + c_j \leq 1, c_i + c_0 \leq 1, c_0 \leq c_i (i, j \in \{1, \dots, m\})\}$  and  $cNE(C_{AE}) = (1/2, 1/2, \dots, 1/2, 1/2)$  in symmetric case with maximum costs exists if and

only if

$$\begin{aligned}
\int_0^{1/a} u_0(p)(1-p)dF(p) &\geq \int_0^1 u_0(p)(1-p)^2 dF(p), \\
\int_0^{1/a} u_0(p)(1-p)dF(p) &\geq \int_0^{1-\sqrt{1-\frac{1}{a}}} u_0(p)dF(p), \\
\frac{2}{m} \int_{1/a}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) + \frac{1}{m} \int_0^{1/a} u(p)pdF(p) &\geq \\
&\geq \max\left\{\int_0^1 u(p) \frac{1-(1-p)^2}{2} dF(p), 0\right\}, \\
\int_{1/a}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) &\geq 0.
\end{aligned}$$

**Proposition 5.** Exploding Offer Equilibrium with  $C_{EO} = \{c : c_1 + c_2 + \dots + c_m \leq 1, c_i + c_0 > 1, c_i \leq c_0 (i \in \{1, \dots, m\})\}$  and  $cNE(C_{EO}) = (1/m, 1/m, \dots, 1/m, 1)$  in symmetric case with maximum costs exists if and only if

$$\begin{aligned}
\int_0^{1-\sqrt[m]{1-\frac{1}{a}}} u_0(p)dF(p) &\geq \int_0^{1-\sqrt[t]{1-\frac{1}{a}}} u_0(p)(1-p)^{m-t}dF(p), \\
&\quad (t = 0, 1, \dots, m-1), \\
\int_{1-\sqrt[m]{1-\frac{1}{a}}}^1 u(p) \frac{1-(1-p)^m}{m} dF(p) &\geq 0, \\
\int_{1-\sqrt[m]{1-\frac{1}{a}}}^1 u(p) \frac{1-(1-p)^m}{m} dF(p) &\geq \\
&\geq \frac{m-1}{m} \int_{1-\sqrt[m]{1-\frac{1}{a}}}^1 u(p) \frac{1-(1-p)^{m-1}}{m-1} dF(p), \\
\int_{1-\sqrt[m]{1-\frac{1}{a}}}^1 u(p) \frac{1-(1-p)^m}{m} dF(p) &\geq \\
&\geq \int_{1-\sqrt[m-1]{1-\frac{1}{a}}}^1 u(p) \frac{1-(1-p)^m}{m} dF(p) + \int_0^{1-\sqrt[m-1]{1-\frac{1}{a}}} u(p)pdF(p).
\end{aligned}$$

## 6 Conclusion

We consider a model of two-sided market where agents on one side (firms) set their application costs, and agents on the other side (workers) apply restricted by their limited budget. These costs serve as a screening instrument that allows higher-ranked firms to regulate number and quality of workers. Also, the case when firms can change their threshold of acceptance is considered. We find the optimal value of this threshold in the simplest case of two firms.

The model reasonably describes a market of prospective PhD students giving possible insights concerning existing equilibria in North American and European schools. We also get new information about some other equilibria that appear in our model, e.g. exploding offers or single applications. In particular, we show that exploding offers are beneficial not only for a weaker university but also for a better school: the latter deals with a higher-quality pool of candidates. European equilibrium may be driven by more positive evaluation of students and less competitive environment.

## Appendix

**Proposition 1.** Let

$$\begin{aligned} U_0^1 &= \int_0^1 u_0(\theta)G(\bar{x} - \theta)dF(\theta), & U_0^2 &= \int_0^{\hat{\theta}} u_0(\theta)dF(\theta), \\ U_1^1 &= \int_0^1 u_1(\theta)(1 - G(\bar{x} - \theta))dF(\theta), & U_1^2 &= \int_{\hat{\theta}}^1 u_1(\theta)dF(\theta). \end{aligned}$$

Then the game on Fig. 2 has the following set of (subgame perfect) Nash equilibria ( $0 \leq c_1 \leq 1$ ,  $0 \leq c_0 \leq 1$ ):

1. if  $U_0^1 > U_0^2$ ,  $U_1^1 > U_1^2$ ,  $U_1^1 \geq 0$ , then the NE set is  $(c_1, c_0)$ ,  $c_1 + c_0 \leq 1$  with cNE  $(1/2, 1/2)$
2. if  $U_0^1 > U_0^2$ ,  $U_1^1 < U_1^2$ ,  $U_1^1 \geq 0$ , then the NE set is  $(c_1, 0)$  with cNE  $(1, 0)$
3. if  $U_0^1 > U_0^2$ ,  $U_1^1 < 0$ ,  $U_1^2 \geq 0$ , then the only NE is  $(1 + \delta, 0)$
4. if  $U_0^1 < U_0^2$ ,  $U_1^1 < U_1^2$ ,  $U_1^2 \geq 0$ , then the NE set is  $(c_1, c_0)$ ,  $c_1 + c_0 > 1$  with cNE  $(1, 1)$
5. if  $U_0^1 < U_0^2$ ,  $U_1^1 > U_1^2$ ,  $U_1^1 \geq 0$ , then the NE set is  $(0, c_0)$  with cNE  $(0, 1)$
6. if  $U_1^1 < 0$ ,  $U_1^2 < 0$ , then the NE set is  $(1 + \delta, c_1)$  with cNE  $(1 + \delta, 1)$ .

**Proof.** To be updated. ■

**Proposition 2.** The overall expected utilities of all the firms in the game on Fig. 3 are the following.

**I** Utility for  $V_2$  (we can get the same for  $V_1$  simply by interchanging costs' subindices  $1 \leftrightarrow 2$ )

- If  $c_1 + c_0 \leq 1$  and  $c_1 \geq c_0$  then

$$U = \begin{cases} \int_0^1 u(p) \frac{1-(1-p)^2}{2} dF(p) & c_2 + c_1 + c_0 \leq 1, \\ \int_{\frac{1}{a}}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) + \frac{1}{2} \int_0^{\frac{1}{a}} u(p) p dF(p) & \sum c_i > 1, c_2 + c_1 \leq 1, \\ \frac{1}{2} \int_0^1 u(p) p dF(p) & c_2 + c_1 > 1, c_2 + c_3 \leq 1, \\ 0 & c_2 + c_3 > 1 \end{cases}$$

- If  $c_1 + c_0 \leq 1$  and  $c_1 < c_0$  then

$$U = \begin{cases} \int_0^1 u(p) \frac{1-(1-p)^2}{2} dF(p) & c_2 + c_1 + c_0 \leq 1, \\ \int_{\frac{1}{a}}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) + \frac{1}{2} \int_0^{\frac{1}{a}} u(p) p dF(p) & \sum c_i > 1, c_2 + c_0 \leq 1, \\ \int_{\frac{1}{a}}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) & c_2 + c_0 > 1, c_2 + c_1 \leq 1, \\ 0 & c_2 + c_1 > 1 \end{cases}$$

- If  $c_1 + c_0 > 1$  and  $c_1 \geq c_0$  then

$$U = \begin{cases} \int_{\frac{1}{a}}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) + \int_0^{\frac{1}{a}} u(p) p dF(p) & c_2 + c_1 \leq 1, \\ \int_0^{\frac{1}{a}} u(p) p dF(p) & c_2 + c_1 > 1, c_2 + c_3 \leq 1, \\ \frac{1}{2} \int_{\frac{1}{a}}^1 u(p) p dF(p) & c_2 + c_3 > 1, c_2 \leq 1, \\ 0 & c_2 > 1 \end{cases}$$

- If  $c_1 + c_0 > 1$  and  $c_1 < c_0$  then

$$U = \begin{cases} \int_{\frac{1}{a}}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) + \int_0^{\frac{1}{a}} u(p) p dF(p) & c_2 + c_0 \leq 1, \\ \int_{1-\sqrt{1-\frac{1}{a}}}^1 u(p) \frac{1-(1-p)^2}{2} dF(p) & c_2 + c_0 > 1, c_2 + c_1 \leq 1, \\ \frac{1}{2} \int_{\frac{1}{a}}^1 u(p) p dF(p) & c_2 + c_1 > 1, c_2 \leq 1, \\ 0 & c_2 > 1. \end{cases}$$

**II** Utility for  $V_0$

- If  $c_1 + c_2 \leq 1$  then

$$U_0 = \begin{cases} \int_0^1 u_0(p)(1-p)^2 dF(p) & c_0 + c_1 + c_2 \leq 1, \\ \int_0^{\frac{1}{a}} u_0(p)(1-p) dF(p) & c_0 + c_1 + c_2 > 1, c_0 + \min\{c_1, c_2\} \leq 1, \\ \int_0^{1-\sqrt{1-\frac{1}{a}}} u_0(p) dF(p) & c_0 + \min\{c_1, c_2\} > 1, c_0 \leq 1, \\ 0 & c_0 > 1 \end{cases}$$

- If  $c_1 + c_2 > 1$  and  $\min\{c_1, c_2\} \leq 1$  then

$$U_0 = \begin{cases} \int_0^1 u_0(p)(1-p) dF(p) & c_0 + \min\{c_1, c_2\} \leq 1, \\ \int_0^{\frac{1}{a}} u_0(p) dF(p) & c_0 + \min\{c_1, c_2\} > 1, c_0 \leq 1, \\ 0 & c_0 > 1 \end{cases}$$

- If  $\min\{c_1, c_2\} > 1$  then

$$U_0 = \begin{cases} \int_0^1 u_0(p) dF(p) & c_0 \leq 1, \\ 0 & c_0 > 1. \end{cases}$$

## References

- [1] H. Chade, L. Smith (2006). Simultaneous Search. *Econometrica*, 74(5), 1293–1307
- [2] D. Gale, L. S. Shapley (1962). College admissions and the stability of marriage. *American mathematical monthly*, 9–15.
- [3] S. Pan (2018). Exploding Offers and Unraveling in Two-Sided Matching Markets. *International Journal of Game Theory*, 47 (1), 351–73.
- [4] A. Rees-Jones, R. Shorrer, C. Tergiman (2019). Correlation Neglect in Student-to-School Matching. Working paper
- [5] A. Roth, M. A O. Sotomayor (1992). *Two-sided matching: A study in game-theoretic modeling and analysis*. Cambridge University Press.