Storage games

Sergei Balakin* and Guillaume Roger*

December 2023

Abstract

We study a long-horizon, oligopolistic market with random shocks to demand that can be arbitraged by two large storage operators with finite capacity. The application we speak to is electricity but our results extend to any storable commodity – that is, most commodities. Because the arbitrage spread is so sensitive to market power, storage operators face strong incentives to restrain quantities by tacitly colluding. This cooperation takes new forms thanks to the multiplicity of actions they must take: selling, buying or both. We construct payoff-maximizing equilibria of this stochastic game, and uncover a new form of *Partial Cooperation* that trades off quantities and delay. Head-on competition is not always an equilibrium of the long-horizon game, unlike many standard games, when market power becomes large enough. We present some robustness checks. We also draw implications for policy and suggest poorly competitive storage is a negative externality to the development of the underlying commodity – for example, renewable energy.

Keywords: *stochastic game, dynamic trading, storage*

JEL: *C73, D43, D47, Q41, Q42*

*Monash University. Corresponding author: Roger – guillaume.roger72@gmail.com. Funding from ARENA under grant 2021/ARP016 gratefully acknowledged. We thank George Mailath for his helpful comments.

1 Introduction

In the early afternoon of April 30th 2022, the state of California briefly met all its electricity demand with renewable energy; it even exported some surplus to neighboring states.[1] In the same vein, the state of South Australia produced 120% of its needs in September 2023 – for a few hours.[2] These examples make it plain there is now a pressing need for investment in storage, rather than in renewable generation capacity. Storage is the bottleneck of the energy transition. It is the means of smoothing the production and consumption of energy, which delivers renewable energy when needed rather than when available. This is relevant in other large markets beyond California or Australia – for example, Spain or Texas – and perhaps more importantly in emerging economies that are so poorly served by conventional power sources.

However we know very little of the economics of electricity storage – nor for that matter, the economics of storage in general. This paper contributes, in part, to addressing this gap. We study a long-horizon trading game based on storage in an oligopolistic market. In each period, demand is subject to aggregate shocks, which affords *intertemporal* arbitrage opportunities to two competing storage units. In this environment we are particularly interested in understanding the incentives to engage in cooperative behavior – tacit collusion. We find they are pervasive and rooted in the market power of storage operators. An important consequence of restraining quantities traded is that the market for the underlying commodity is underdeveloped. In the context of the energy transition, this implies that the bottleneck persists.

We make substantive and technical contributions to the storage problem, where our main application is electricity (and so drives much of the language we use). On the substantive front, we reiterate the sensitivity of the arbitrage revenue to market power, that is, almost directly to capacity. Arbitrage revenue is so sensitive because an arbitrageur must sell and buy, both of which are subject to market power. This responsiveness induces strong, indeed

[1] Source: Electrek: https://electrek.co/2022/05/02/california-runs-on-100-clean-energy-for-the-first-time-with-solar-dominating/.

[2] Source: RenewEconomy, https://reneweconomy.com.au/solar-reaches-record-120-per-cent-of-electricity-demand-in-south-australia/

at times overwhelming, incentives to restrain traded quantities, with two consequences. First, head-to-head competition is not always an equilibrium of the dynamic game – in contrast, for example, to a standard Cournot game. Second, the need to restrain quantities develops into a strong incentive to engage in tacit collusion. We determine these incentives in terms of capacity levels.

On the technical front, we construct payoff-maximizing equilibria of this *stochastic* game that are sustained by a grim-trigger strategy.[3] This most extreme form of penalty supports the equilibria with the largest payoffs. The novelty is twofold: first, the form these equilibria take, and second, the precise manner they are supported and constructed. On the first account, a new class of equilibrium that we label "Partial Cooperation" emerges thanks to the multiplicity of actions a storage unit faces. Indeed, storage must both buy and sell to generate its revenue, which enhances the scope of cooperative behavior and is not present in standard models, where selling it the only relevant action. Partial Cooperation arises as a compromise between restraining quantities (through cooperation) and the effect of discounting. This new class of cooperative equilibrium is supported (in part) by a likewise new asymmetric, non-cooperative equilibrium that we dub "Follow-the-Leader". In such an equilibrium, a player accepts to only trade when the stochastic process and the profile of states of charge allow it – and in so doing, side-steps direct competition. For some capacity levels it is the only non-cooperative equilibrium, and therefore the only means to support any cooperation. On the second account, the machinery of repeated games does not directly apply to a stochastic game – for example, the "one-shot deviation principle" (Abreu (1988)) does not strictly hold. One consequence is that even though we consider equilibria from an *ex ante* standpoint, the exact construction hinges on the incentives of players conditional on the state variable (of the continuation value) at the *interim* stage. This makes matters rich and at times delicate.

Our work applies to other commodities, such as agricultural commodities, fuels or others. It can be made relevant to each of these by adjusting the efficiency parameter and the discount factor, with different effects on equilibrium outcomes. A higher discount rate that

[3] For clarity, there is a unique payoff-maximizing equilibrium for each capacity level; we construct each equilibrium for a range of capacities.

corresponds to high-frequency trading such as electricity, for example, favours cooperation as an equilibrium. With a lower discount rate, as for crops, Partial Cooperation becomes more attractive. The model can also extend to market making in securities.[4]

Much to our surprise the more general "storage problem" is not well understood yet, despite storage being used for millennia and some prior papers tackling the question. The present model features two essential characteristics that are (jointly) absent in any other paper: a stochastic environment and market power; the latter is central. In electricity, Andres-Cerezo and Fabra (2023b) study the question of market structure with storage, but leave aside how storage actually behaves. Andres-Cerezo and Fabra (2023a) present a model of cyclical storage and correlated renewable generation, in which all parties are price takers. This renders the dynamics moot: without price impact, storage charges to capacity and discharges in full every cycle; therefore it is enough to analyse a single cycle. In our model, absent market power there are no incentives to restrain quantities and thus no need to collude. Butters et al. (Working Paper) use California data to estimate the equilibrium effect of large-scale storage. In that model however storage is assumed to behave competitively; this is almost orthogonal to our work. Karaduman (2020) is the first to study grid scale storage. He does allow for market power; however he does not compute the best reply but simulates it using Australian data. Therefore the actual behavior of the storage unit remains unknown. Williams and Green (2022) compute the welfare effects of storage on the current British market using simulations, and so without characterising any equilibrium, nor with uncertainty. Geske and Green (2020) do study arbitrage in a model of imperfect competition with demand uncertainty and diurnal, weekly and seasonal patterns. They must confine themselves to numerical (approximate) solutions to the welfare maximization problem, and show quantity withholding. We construct subgame-perfect equilibria of the game.

Hydro-electric power differs from storage. Once a dam is built, the water inflow is free, exogenous and stochastic; in contrast, a storage unit pays for the energy it buys, it can have (a measure of) monopsony power, and it makes that decision optimally as part of its trading

[4]Indeed, this intermediation activity shares many characteristics with trading electricity through storage: assets are bought and sold, a revenue is generated by arbitrage, holding inventory is necessary and price impact matters a great deal. Market making differs from idiosyncratic trading (e.g. Vayanos (1999)).

strategy. In addition, most models of dam management take prices as *fixed* (not even moving in the aggregate) and so reduce to an optimal control problem. In the inventory management problem (see Harrison and Taylor (1978) for example), the problem is strictly one of stochastic control – not a game – in which the per-unit payoffs (rewards or costs) are exogenous.

Further afield, Deaton and Laroque (1992) rationalise the volatility of 13 essential crops by introducing *perfectly competitive* storage in a model of speculative trading. Wright and Williams (1984) conduct a welfare analysis of the benefits of commodity storage, in which, again, all parties are price takers. In addition there are no actual dynamics in that model. Even Samuelson (1971) dabbles in the problem, however still under the assumption of perfect competition. Hence the simple intermediation exercise (buying and selling) over a long horizon in a strategic environment demands more of our attention.

While conceptually a model of arbitrage, our work departs from that rich literature (e.g. Dávila et al. (2024), Shleifer and Vishny (1997), Oehmke (2009) and many others). In these papers, arbitrage is contemporaneous (across segmented markets), fundamentally riskless and there is no aggregate risk. Here arbitrage can only be intertemporal (in a single market) and risky, and there is aggregate risk in the economy. Dávila et al. (2024) show the marginal social value of arbitrage is the price gap between arbitraged securities. Such a measure is not applicable here because of aggregate risk. Gromb and Vayanos (2002) allow for risky arbitrage and show there is either too little or too much risk-taking in equilibrium (compared to the social optimum). In our model, quantities are systematically restrained.

2 Model

Consider a market with two storage units, n producers (electricity generators) labelled $j = 1, 2, ...n$, and a pool of consumers. Retailers and consumers are confounded and retailing has no cost; equivalently, retailers have no market power and perfectly reflect the behavior of consumers. That behavior is described by the demand function $D(p_t, \varepsilon_t)$ for each period

t, where ε_t is a shock distributed according to some commonly known distribution F.[5] Each producer j offers a quantity q_t^j for each period t; we ignore capacity constraints on the producers (generators).[6] The storage units are identical and have finite capacity k. In each period, a storage unit can either buy (charge) $b_t^i \geqslant 0$ up to its capacity, or sell any quantity $s_t^i \geqslant 0$ (discharge).[7] A storage operator can only either buy or sell in each period, so $b_t^i \cdot s_t^i = 0$ for any t – this is a technical characteristic.[8] For each unit $i = 1, 2$, this process gives rise to a simple equation of motion:

$$c_t^i = c_{t-1}^i + b_t^i - \frac{s_t^i}{\delta}, \qquad t \in \mathbb{N}, \qquad c_0^i = 0. \tag{1}$$

The quantity c_t^i is a current level of charge ($0 \leqslant c_t \leqslant k$) and δ is a round-trip efficiency parameter ($0 < \delta \leqslant 1$) that applies to both units. We suppose the storage units face a discount factor $\beta < 1$; they are exposed to a strictly positive interest rate. The parameters δ and β play a different role. Roughly speaking δ captures the economic viability of storage, while β represents the more standard patience (or an interest rate); δ can also be interpreted as a marginal cost. More precisely, these two parameters actually interact in the payoffs and jointly determine the incentives of the players; together they are the reason behind Proposition 5, in particular. The market clears if

$$D(p_t, \varepsilon_t) = \sum_{j=1}^{n} q_t^j + \sum_{i=1}^{2} [s_t^i - b_t^i]$$

[5] One can also add renewable energy with stochastic supply and conceive of the demand function as residual demand without material consequences.

[6] The norm in electricity markets it to use the more elegant supply-function equilibrium (SFE); however the richness of the SFE is lost here since we rely throughout on binary shocks; see Klemperer and Meyer (1989). Further, the Cournot outcome is a possible equilibrium outcome of the SFE and constitutes an upper bound for the payoffs to suppliers (Klemperer and Meyer (1989)). Finally, quantity competition is used as a successful proxy in many papers (Acemoglu et al. (2017), Willems et al. (2009), Lundin and Tangerås (2020)); much of this work relies on the estimations of Borenstein and Bushnell (1999), Borenstein et al. (1999) or Bushnell et al. (2008).

[7] We make no distinction between power and energy; it is as if a quantity were either energy or power for a prescribed duration (e.g. for the trading interval).

[8] However it is also clear that it cannot be optimal to charge and discharge simultaneously.

for any t. Since the nature of competition is not the primary object of interest we consider a linear demand function:

$$D(p_t, \varepsilon_t) = 1 - p_t + \varepsilon_t.$$

Throughout the rest of the paper, we assume that the shocks ε_t, $\varepsilon \in \mathcal{A} := \{-a, a\}$ are independently and identically distributed,

$$Pr\{\varepsilon = a\} = Pr\{\varepsilon = -a\} = 1/2, \qquad 0 < a < 1 \tag{2}$$

for any t. Depending on the decisions of the storage operators, in each round there may be n (symmetric) competitors, $n+1$ competitors, with the active storage unit having a limited capacity, or $n+2$ competitors – two of which being capacity constrained. Our goal is to construct subgame-perfect Nash equilibria (SPNE) of this stochastic game. We dispense with folk theorems to focus on the behaviour of players instead. There are many such SNPEs, therefore we focus on equilibria that (a) maximise the payoffs to storage units and (b) (consequently) may feature some form of cooperation. As is standard, side transfers are assumed to not be feasible.

Some of these equilibria are also Markov-perfect equilibria (MPE). One could argue, given the stochastic environment, that the concept of Markov-perfection is more "natural" in the sense of less taxing on players. It is certainly less taxing on the analyst, and lends itself to computation. Where the MPE differs from the SPNE, it is not onerous to identify it using the same techniques we employ.

Characterizing equilibria for arbitrary policies $\{b_t^i, s_t^i\}_{t=0}^{\infty}$, $i = 1, 2$ is an impossible task. Hence, most of our analysis focuses on the case, in which storage operators are restricted to charge and discharge in full: $b^i, s^i \in \{0, k\}$. Limiting attention to binary actions is common in much of the literature on dynamic games or repeated games. This restriction does induce some rigidity that renders cooperation more attractive than if we allowed for a more flexible play.[9] We do provide some results on more flexible actions and confirm this observation, but also our results. Flexibility and cooperation are substitutes, but imperfect ones; our results remain relevant and our insights valid.

3 Competing, colluding and letting go

Even if restricting the strategy space so that storage units effectively face a binary action space, the game described in Section 2 admits a large number of equilibria, and a general characterisation remains elusive. To make progress in this problem, we reduce the space of admissible strategies in two ways. First, we must describe the equilibrium behavior of the other n players (the producers); we elect to restrict attention to the repetition of the Cournot equilibrium of the stage game. This equilibrium is simple to describe, unlike any of the more sophisticated equilibria one can construct. This choice is further justified by the work of Bonatti et al. (2017), who study a dynamic Cournot model under incomplete information with learning. The equilibrium converges to the repeated static Nash equilibrium. We start from this point.

Second, we stay true to form in that, where cooperative play is concerned, we construct equilibria that maximise the surplus of the storage operators, given that the n producers repeat the Cournot stage game. In light of the rigidity of the strategy space of the storage units, this is not particularly controversial. If a unit can only buy or sell in full, the scope of cooperation is limited to the *timing* of actions rather than their magnitude anyway. That is, there is no sense in which storage units can distort their own quantities, but only their *aggregate* quantity; they do so by taking turns in trading–this is reminiscent of bidding rings in the auction literature. Therefore, the best-reply choices of the n producers remain unchanged as well; if playing Cournot is a best response before storage units collude, it is one after they do. Rather the point of selecting equilibria that maximise payoffs to the storage units is that they are supported by the most extreme off-equilibrium punishments, which are simple

[9] That is, unable to soften competition by unilaterally reducing quantities, storage operators engage in cooperative play instead.

to describe. In addition, these payoffs are part of the extremal payoffs that characterise the entire payoff set that is achievable through some form of cooperation.

3.1 Behaviors and payoffs in the stochastic game.

The objective of a storage operator is

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t p_t(s_t^i - b_t^i)\right], \quad i = 1, 2, \tag{3}$$

to be maximized subject to the law of motion (1), the capacity constraint

$$0 \leqslant c^i \leqslant k, \quad i = 1, 2 \tag{4}$$

and where the price p_t is a function of the aggregate quantity $Q_t = \sum_j^n q_t^j + \sum_i^2 (s_t^i - b_t^i)$. The state variables of this problem are a pair of states of charge and demand shocks $(\mathbf{c}, \varepsilon) \in \mathcal{C} \times \mathcal{A}$, so actions are mappings $b_t^i, s_t^i : \mathcal{C} \times \mathcal{A} \times \mathcal{H}_t \mapsto \{0, k\}$ for each i, where $\mathcal{H}_t$ is the set of all histories up to time t with $\mathcal{H}_0 = \emptyset$. Hence the continuation game need not be a replica of the current stage game. Because actions $b^i(\mathbf{c}_t, \varepsilon; H_t)$ and $s^i(\mathbf{c}_t, \varepsilon; H_t)$ already encode the state $(\mathbf{c}, \varepsilon)$ of the system, histories $H_t \in \mathcal{H}_t$ are constructed in standard fashion. For each player i, a strategy is a sequence of actions from time 0 to ∞ : $\{(b_t^i, s_t^i)\}_{t=0}^{\infty}$. The corresponding value function to player i writes

$$\forall i, \quad V^i(\mathbf{c}, \epsilon) = \sup_{b^i, s^i} \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t p_t(s_t^i - b_t^i)\right], \text{[10]} \tag{5}$$

A subgame-perfect Nash equilibrium of this game is a profile of strategies $(\hat{\mathbf{b}}_t, \hat{\mathbf{s}}_t)$ such that (5) holds for each agent i, given agent j's equilibrium strategy $(\hat{b}_t^j, \hat{s}_t^j)$. To ease notation we drop ε from the functions V^i as it is obvious. It is important to bear in mind that $\forall i,\ c_0^i = 0$; charging is the first action a storage operator can take.

For convenience, we define charging costs (when purchasing) under the negative shock as

[10] We dispense proving that the Dynamic Programming Principle holds in this environment, which is quite standard.

B_l if l storage units buy, and the revenue a storage unit earns when selling under the positive shock as A_l (again, depending on number of storage units l selling in that period):

$$B_l = \frac{1 - a + lk}{n + 1} \cdot k, \qquad A_l = \frac{1 + a - l\delta k}{n + 1} \cdot \delta k, \qquad l \in \{1, 2\}.$$

In both expressions, the right multiplier (k or δk) is just a quantity bought or sold. The left multiplier is the resulting Cournot price. It is easy to check that it is optimal to charge only when the shock ε is positive, and likewise to discharge when the shock is negative.[11] Then we are left with n generators competing for the residual demand, which is $1 - a + lk$ or $1 + a - l\delta k$ for negative and positive shocks, respectively.

Two characteristics are new in this work and lead to novel results. First, storage units must charge and discharge to generate any payoff. There being two (sets of two) actions allows more flexibility in the scope of cooperative play. When firms only sell, they need only decide whether to cooperate on selling. Here, they have to decide whether to cooperate when buying, selling or both. Cooperation means taking turns since they can only buy or sell in full. Because this is a stochastic game (and not a repeated game), *when* each can take turns depends on the combination of the realizations of the stochastic process and the play. This feeds into the incentive constraints an equilibrium must satisfy. Second, the *arbitrage spread* $A_l - B_{l'}$ (where l need not be equal to l') is the source of all surplus and is very sensitive to market power.[12] Indeed, the function A_l starts at zero, is concave and reaches a local maximum at $k = (1 + a)/(2l\delta)$; B_l also starts at zero, is convex and monotone increasing. Therefore the spread is a concave function of the capacity k with an interior maximum, and rapidly reaches zero for large k. Hence whether to collude, and the manner in which cooperation is implemented, depend greatly on the market power of storage – that is, on the capacity of storage units.

To begin with, we provide a list of possible behaviors that may emerge as equilibria; not all these are equilibria – this analysis is forthcoming. This list need not be exhaustive in the

[11] Hence, in terms of notation, there is no loss in dispensing with the shock ε as a state variable.

[12] l need not be equal to l' because, for example, one unit may charge at t but both of them may discharge at $t + 1$.

class of behaviors that can be supported as equilibrium by a grim-trigger strategy; rather they deliver upper bounds on the payoffs that can be achieved. Throughout we distinguish *behavior*–all in lower case–from *equilibrium*, which takes an uppercase.

1. *competition.* There is no coordination at all. Empty storage units always buy when they face negative shocks. Full storage units always sell when they face positive shocks. Both stay idle otherwise.

2. *partial cooperation.* Storage units coordinate on buying but not on selling. That is, if both storage units face negative shocks while empty, they flip a coin to decide who is first to buy; the losing party stays idle. If both storage units face positive shocks while full, they sell simultaneously (i.e. without coordination). If the storage units have different states of charge, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

3. *partial cooperation alt.* Storage units coordinate on selling but not on buying. If both storage units face negative shocks while empty, they buy simultaneously (no coordination). If both storage units face positive shocks while full, they flip a coin to decide who sells first; the losing unit stays idle. If the storage units have different states of charge, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

4. *cooperation.* Storage operators coordinate on both buying and selling. If both storage units face negative shocks while empty, they flip a coin to decide who is first to buy; the losing party stays idle. If both storage units face positive shocks while full, they flip a coin again to decide who sells first; the losing unit stays idle. When they have different states of charge, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

5. *follow the leader (ftl).* One of the storage units (the "leader") always buys (sells) first when both units are empty (full) under a negative (positive) shock. The second one (the "follower") stays idle. If they have different states of charge, the empty one always buys

under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise. This is an asymmetric, non-cooperative play. Coordination is required, but not cooperation.

6. *ftl+competition.* The leader always buys first when both units are empty under a negative shock. The follower stays idle. If both storage units face positive shocks while full, they sell simultaneously (no coordination). If the storage units have different states of charge, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

7. *competition+ftl.* If both storage units are facing negative shocks while empty, they buy simultaneously (no coordination). When both units are full, the leader always sells first under a positive shock. The follower stays idle. If the storage units have different states of charge, the empty one always buys under a negative shock, and the full one always sells under a positive shock. They stay idle otherwise.

The first four behaviors deliver symmetric (expected) payoffs, while the last three do not. Because mostly of efficiency losses $\delta \leqslant 1$, behaviors 3 and 7 are always (payoff-) dominated by 2 and 6, respectively. They are discarded from now on. Next we compute the payoff function that is induced by each of these behaviors to understand their desirability, as well as their capacity to either become an equilibrium, or to support an equilibrium. That is, we can write recursive equations of the form

$$\mathbf{V}(\mathbf{c^1}, \mathbf{c^2}) = \mathbf{P} + \beta\mathbf{Q}\mathbf{V}(\mathbf{c^1}, \mathbf{c^2}), \tag{6}$$

that correspond to each of these behaviors, and compute them in terms of B_l, A_l and the discount factor β.[13] In Equation (6) $\mathbf{c^i}$ are vectors, $\mathbf{V}$ is a vector of continuation values, $\mathbf{P}$ a vector of flow payoffs and $\mathbf{Q}$ a square matrix. These results are collated in our first Proposition, and each of these payoff functions are also drawn in Figure 1.

Proposition 1. *The payoffs for the equilibrium-candidate behaviors are:*

[13] That is, as functions of a, k and β.

- *competition:*

$$U_{com} = -\frac{B_2}{2} + \frac{\beta}{4(1-\beta)}\left(A_2 - B_2\right).$$

- *partial cooperation:*

$$U_{pc} = -\frac{2+\beta}{8}B_1 + \frac{\beta}{16(1-\beta)}\Big(\left(2-\beta\right)\left(A_1 - B_1\right) + 2\beta\left(A_2 - B_1\right)\Big).$$

- *cooperation:*

$$U_{col} = \frac{1}{2(2-\beta)}\left(-B_1 + \frac{\beta}{(1-\beta)(2+\beta)}\left(A_1 - B_1\right)\right).$$

- *ftl:*

  The leader's payoff:

$$\bar{U}_{ftl} = -\frac{B_1}{2} + \frac{\beta}{4(1-\beta)}\left(A_1 - B_1\right).$$

  The follower's payoff:

$$U_{ftl} = \frac{\beta}{2(2-\beta)}\left(-B_1 + \frac{\beta^2}{2(1-\beta)(2+\beta)}\left(A_1 - B_1\right)\right).$$

- *ftl+competition:*

  The leader's payoff:

$$\bar{U}_{fc} = -\frac{B_1}{2} + \frac{\beta}{8(1-\beta)}\Big(\left(2-\beta\right)\left(A_1 - B_1\right) + \beta\left(A_2 - B_1\right)\Big).$$

  The follower's payoff:

$$U_{fc} = \frac{\beta}{4}\left(-B_1 + \frac{\beta}{2(1-\beta)}\left(A_2 - B_1\right)\right).$$

These functions are linear in the terms A_l and B_l, and therefore are quadratic functions of capacity k. The arbitrage spread $A - B$ may depend on the number $l \in \{1, 2\}$ of firms buying or selling at any time t. This spread is then discounted by the factor β, not always symmetrically depending on the behavior, and net of the initial charge. For example, under competition, everything is symmetric and $l = 2$ always. The first term is the first charge, which occurs at time zero with probability 1/2. The spread is then discounted starting from time $t = 1$, and weighed by 1/4, which is the frequency of a full cycle $(-a, a)$. Under partial cooperation, the spread is made of a linear combination of A_2, A_1 and B_1, which are discounted a different rate since they do not occur with the same frequency, and the initial charge is B_1 rather than B_2 since they charge sequentially. We graph these payoff functions (for one storage unit) in Figure 1.

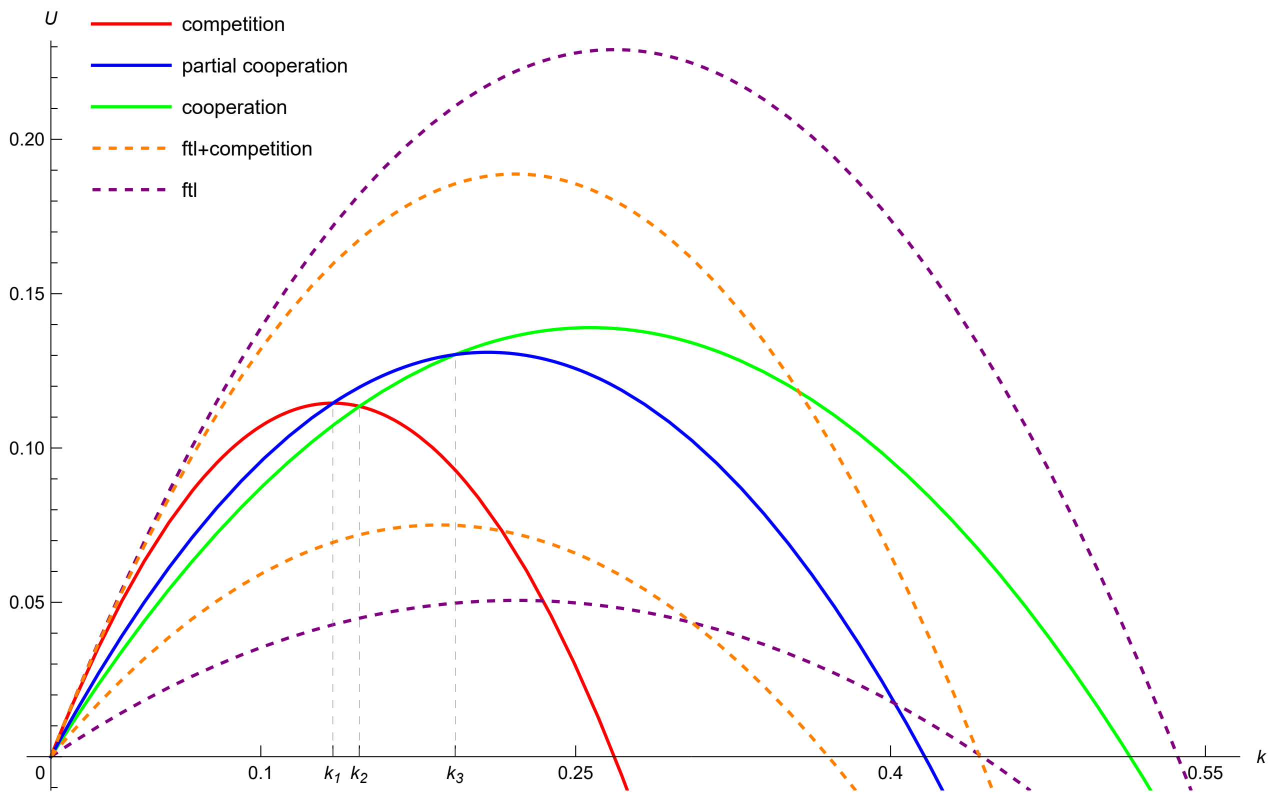

Figure 1: Payoff functions for different strategies as a function of capacity for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

Cooperation (green) is an attractive behavior for large enough a capacity in that its payoff is the largest, but competition is more attractive for low capacities. The dotted lines are payoffs arising from *asymmetric* behaviors – a leader and a follower – and so deliver asymmetric payoffs; the leader always receives larger payoffs.

For behaviors relying on cooperation to exist as an equilibrium, one must find an off-equilibrium punishment to support them. This is far from obvious: for example, for very large capacities, competition is *not* an equilibrium – it is trivially dominated by $b = s = 0$, for example. Therefore it cannot support cooperation as an equilibrium.[14] This feature is evidence that storage does differ from standard production-for-sale.

Figure 1 suggests that whether a behavior can be sustained as an equilibrium depends on the storage capacity k. Hence we introduce three of many thresholds we need in our analysis. They are labelled k_1 to k_3 and denote the capacity level at which a storage operator is indifferent *in terms of payoffs* between any two behaviors. k_1 is that capacity level such that the payoffs to competition and partial cooperation are the same; k_2 identifies indifference between competition and cooperation, and k_3 between partial cooperation and cooperation. Throughout we require a condition that connects the magnitude of the shocks to the technical parameters β and δ. This magnitude (volatility) must be large enough for storage to have a role to play.

Lemma 2. *If condition*

$$\frac{1-a}{1+a} < \frac{\beta\delta}{2-\beta} \tag{7}$$

holds, then there exist thresholds $0 < k_1 < k_2 < k_3$.

Before studying which of the listed behaviors are, or not, an equilibrium, the following observation complements Proposition 1 and is a precursor to the main result.

Remark 3. *A single firm owning both storage units receives payoffs that are the upper envelope of the payoff functions listed in Proposition 1. It adopts the corresponding behavior depending on whether* $k < k_1$, $k_1 < k < k_2$ *or* $k > k_2$. *It may adopt a symmetric or asymmetric behavior, which results in the same aggregate payoffs.*

The single entity need not be concerned with equilibrium; it can commit to itself, internalize all conflicts and implement "frictionless cooperation". Instead, competing firms must satisfy

[14] It soon becomes apparent why competition is not always an equilibrium, while it is always in the repeated Cournot game, for example.

incentive constraints when they are relevant, which depends on the state of the game.[15] Hence the upper envelope of our equilibria differs from Figure 1.

The forthcoming results are expressed in terms of capacity k; from the definitions of A and B and from Proposition 1, it is clearly the essential characteristic. Not all equilibria are supported by the same punishment threat. The description of these equilibria is subtle at times; we pare down the details and notation as much as possible and relegate these to the Proofs. To ease exposition we break down this exercise in three parts.

3.2 Competing and letting go

This first case is the most natural one, yet not immediately intuitive.

Proposition 4. *Suppose Condition (7) holds, then Competition is a subgame-perfect Nash equilibrium for* $0 < k < \overline{\kappa}_r$*, where*

$$\overline{\kappa}_r = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{4 - \beta + 2\beta\delta^2} > 0.$$

When capacity k is small enough, the symmetric non-cooperative play is an equilibrium; it ceases to be an equilibrium as soon as capacity exceeds $\overline{\kappa}_r$.[16] In this case, one of the players is better off *letting go* and following the other one – the leader – under the ftl+competition behavior. The reason is that capacity becomes large enough to erode the arbitrage spread; this erosion is so acute from $\overline{\kappa}_r$ on that even the follower is better off under this new regime. The threshold $\overline{\kappa}_r$ is the point of indifference for the follower. At k_4 the payoff-maximizing non-cooperative behavior switches from ftl+competition to ftl, and at k_p from ftl to zero; that is, the follower stops being active. These equilibrium payoffs and the relevant thresholds are depicted in Figure 2.

It is perhaps surprising that competition is not always an equilibrium; after all, it is an

[15] In addition, transfers cannot be used to relax these constraints.

[16] In the proof we show $\overline{\kappa}_r$ to be the smallest of many thresholds that correspond to many possible deviations that may combine both buying and selling. The multiplicity of actions not only increases the scope for deviations, it also renders deviations sensitive to the exact continuation play. This is not the case in a standard repeated game.

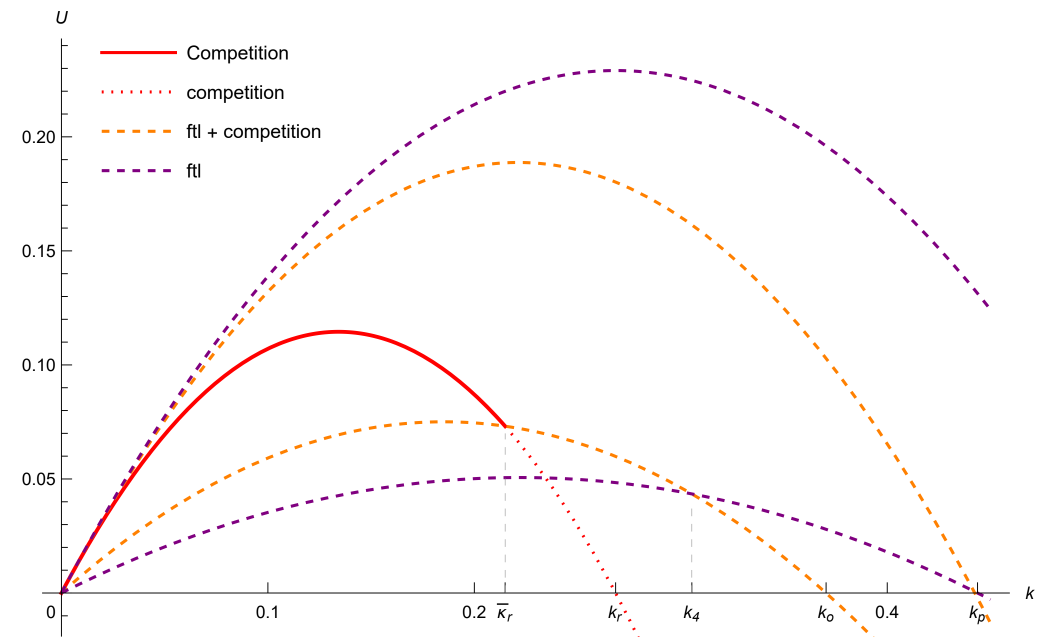

Figure 2: Non-cooperative payoff functions and Competition payoffs as a function of capacity for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

equilibrium in the repetition of the Cournot game. This probably best highlights the profound difference between "the storage game" and conventional production-for-sale. When only selling, players only need to care about $A(l, k)$. But a storage unit also exerts market power when *buying*; the function $B(l, k)$ is increasing in both arguments and convex in k. Selling "too much" by imperfectly internalising one's infra-marginal impact also requires buying "too much", which very rapidly destroys any arbitrage revenue. Then letting go dominates.

To be clear, follow-the-leader requires no cooperation at all; it is an asymmetric, non-cooperative, coordination behavior of this game that can be played repeatedly without any supporting penalty regime. It can produce strictly positive payoffs for the follower because of the combination of uncertainty and capacity constraint: once the leader has moved and the next shock is in the same direction (e.g. negative), then the follower can move (e.g. buy), and wait for the opportunity to sell later. This characterization is important for another reason: this behavior can be used to support cooperative play.[17]

[17] At this point we do not claim that ftl is an equilibrium; it is an appealing suggestion but we do not check whether it is immune from deviation. Instead we focus on symmetric equilibria. Later we do check that ftl and its variations are a SPNE for some k when constructing cooperative equilibria.

3.3 Competition, Partial Cooperation – and letting go

Partial cooperation is one such cooperative play. As we know since the work of Abreu (1988), a cooperative equilibrium of a repeated game can be supported with simple penal codes; here, the simple penal code is reversion to any non-cooperative equilibrium – for example, Competition, but not exclusively. For emphasis, this is a stochastic game, so the methods of Abreu (1988) must be adapted as needed.

Proposition 5. *Assume that (7) holds. Let*

$$\underline{\kappa}_b = \frac{(8-8\beta+\beta^2)(-(2-\beta)(1-a)+\beta(1+a)\delta)}{32-40\beta+14\beta^2-\beta^3+(16\beta-16\beta^2+3\beta^3)\delta^2},$$
$$\overline{\kappa}_b = \min\left\{\frac{-\beta(2+\beta)(1-a)+(8-4\beta-\beta^2)(1+a)\delta}{\beta(2+\beta)+(16-4\beta-3\beta^2)\delta^2}, \frac{(4-\beta)(-(2-\beta)(1-a)+\beta(1+a)\delta)}{8-6\beta+\beta^2+\beta(8-3\beta)\delta^2}\right\},$$

both finite and bounded away from 0. Then for $\underline{\kappa}_b < k < \overline{\kappa}_b$, the following is a subgame-perfect Nash equilibrium that we label Partial Cooperation:

- *The storage units play partial cooperation.*
- *If one of the units deviates by purchasing under a negative shock when both units are empty and it is not its turn, the other unit switches to competitive behavior forever as a punishment. That is, it starts buying (selling) every time it is empty (full) under a favorable shock. A Nash Equilibrium of this subgame off the equilibrium path is as follows:*
  - *both units play Competition if $\underline{\kappa}_b < k < \overline{\kappa}_r$;*
  - *the deviating unit plays FTL+Competition if $\overline{\kappa}_r < k < \min\{k_4, k_o\}$, where*

$$k_4 = \frac{-\beta^2(1-a)+(4-2\beta-\beta^2)(1+a)\delta}{\beta^2+2\left(4-\beta-\beta^2\right)\delta^2}, \quad k_o = \frac{-(2-\beta)(1-a)+\beta(1+a)\delta}{2-\beta+2\beta\delta^2};$$

  - *the deviating unit plays FTL if $k_4 < k_o$ and $k_4 < k < \min\{k_p, \overline{\kappa}_b\}$, where*

$$k_p = \frac{-(4-2\beta-\beta^2)(1-a)+\beta^2(1+a)\delta}{4-2\beta-\beta^2+\beta^2\delta^2};$$

– *the deviating unit discharges (either competitively or using ftl strategy) as soon as possible and exits the market forever afterwards for any remaining $k \leqslant \overline{\kappa}_b$.*

Here the threshold $\underline{\kappa}_b$ is the capacity level such that the incentive to play the Partial Cooperation equilibrium dominates the deviation gain (competing when one should not) and enduring the punishment forever after. As in Proposition 4, above the threshold $\overline{\kappa}_b$ Partial Cooperation can no longer be an equilibrium because its payoff is dominated by either follow-the-leader, or by playing zero if being the second mover. As before, one of the players accepts to be a follower; this is "letting go". Depending on capacity k, partial cooperation can be supported as an equilibrium thanks to the existence of one of the four, non-cooperative plays to which players can revert in case of deviations that are identified in Section 3.2. We show the equilibrium payoff and the range on which the equilibrium exists in Figure 3, where some subtleties demand explaining.

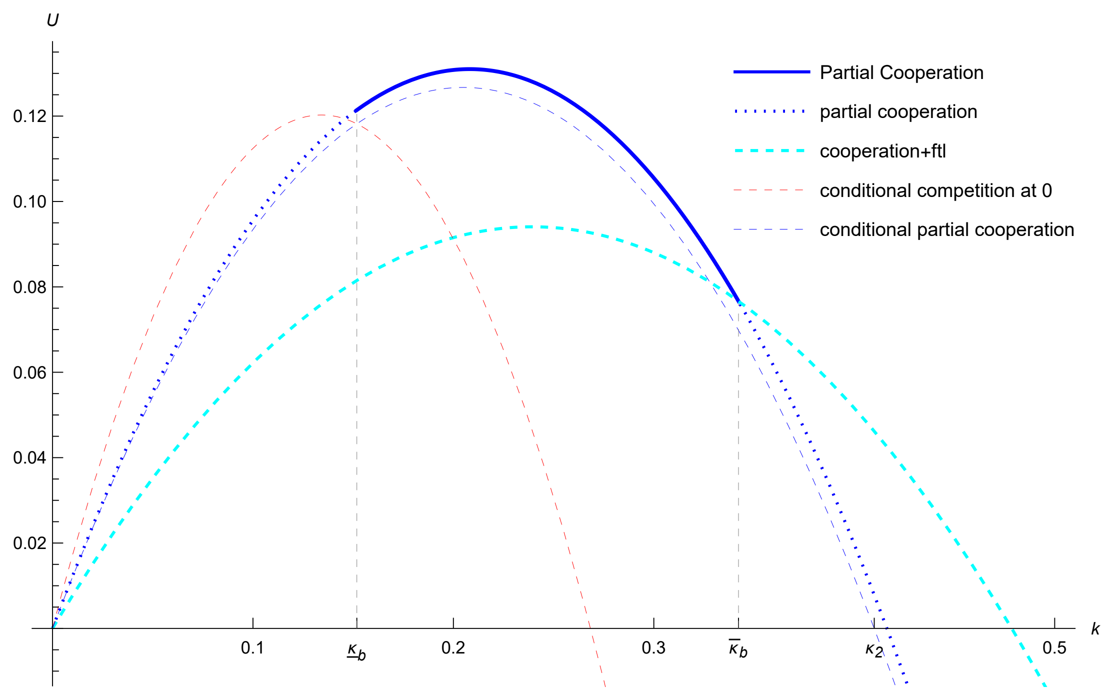

Figure 3: Partial Cooperation payoffs with the payoffs of possible deviations for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

Figure 3 depicts the case where $\overline{\kappa}_b = \frac{-\beta(2+\beta)(1-a)+(8-4\beta-\beta^2)(1+a)\delta}{\beta(2+\beta)+(16-4\beta-3\beta^2)\delta^2}$. The solid blue line represents the *ex ante* expected payoff from Partial Cooperation; the dashed blue line passing through the point κ_2 is the *interim* expected payoff from playing the same equilibrium for the party who moves *second*. A SPNE must be subgame perfect at every node; here it implies it must continue to be an equilibrium for every state of charge c_t^i. However the incentives to cooperate do differ depending on that state of charge: with discounting, the payoff to the first-moving unit is always higher than that to the second-moving unit – on and off the equilibrium path. The *interim* expected payoff, which accrues to the second mover, is the relevant payoff to determine whether the behavior can be sustained as an equilibrium since it is the worse payoff from partial cooperation. That payoff is contrasted to the payoff from competition when in the same state $c_t^i = 0$; the indifference point is the threshold $\underline{\kappa}_b$. That threshold is then applied to the *ex ante* expected payoff (the solid line) to determine the equilibrium. Conversely, the upper bound $\overline{\kappa}_b$ is determined by the incentives of the players *ex ante*. At that point, even the player who is becoming the follower is better off giving up on partial cooperation. Rather than competing when selling, it becomes more profitable to follow the leader when selling too; this is labelled "cooperation+ftl". The intersection with the solid blue line (partial cooperation) identifies $\overline{\kappa}_b$.

When $\overline{\kappa}_b = \frac{(4-\beta)(-(2-\beta)(1-a)+\beta(1+a)\delta)}{8-6\beta+\beta^2+\beta(8-3\beta)\delta^2}$ instead, the solid blue line extends all the way to the threshold κ_2 in Figure 3. Cooperation+ftl exists but is dominated, and partial cooperation is supported as the equilibrium Partial Cooperation by the alternative of selling zero when moving second.

Partial Cooperation, which involves playing partial cooperation all the time, is an equilibrium for capacity levels between $\underline{\kappa}_b$ and $\overline{\kappa}_b$. On this range, there may be other equilibria that involve playing partial cooperation some of the time. For capacities in excess of $\overline{\kappa}_b$, Partial Cooperation cannot be an equilibrium. There may be other cooperative equilibria then.

3.4 Competition, Cooperation – and letting go.

Cooperation is a behavior that becomes increasingly attractive as capacity k increases. It is also supported as an equilibrium by the same non-cooperative equilibria as Partial Cooperation, however with some details that are important to the actual construction of the equilibrium. For this we require

$$\frac{1-a}{1+a} < \frac{G_1(\beta) + G_2(\beta)\delta^2}{G_3(\beta) + G_4(\beta)\delta^2} \cdot \delta, \tag{8}$$

where

$$G_1(\beta) = -16 + 28\beta - 8\beta^2 - \beta^3 - \beta^4, \qquad G_2(\beta) = \beta(1+\beta)(2-\beta)^2,$$
$$G_3(\beta) = \beta(2-\beta)(4-\beta-\beta^2), \qquad G_4(\beta) = 32 - 48\beta + 14\beta^2 + 5\beta^3 - \beta^4.$$

with some explanations. Condition (8) implies (7) as $\beta < 1$.[18] So cooperation is a little more onerous than any other play so far. The reason, as is apparent from the definition of the behavior in Section 3.1, is the delay that is involved. An interpretation one can make is that a larger volatility is required to compensate for the delay incurred by the second-moving unit.

Proposition 6. *Assume that Condition (8) holds. Let*

$$\underline{\kappa}_g = \frac{-\beta(4-\beta-2\beta^2)(1-a) + (8-8\beta-\beta^2+2\beta^3)(1+a)\delta}{\beta(8-\beta-3\beta^2) + (16-12\beta-3\beta^2+3\beta^3)\delta^2},$$
$$\overline{\kappa}_g = \frac{-(4-\beta-\beta^2)(1-a) + \beta(1+\beta)(1+a)\delta}{4-\beta-\beta^2+\beta(1+\beta)\delta^2},$$

which are finite and bounded away from 0. There exists some $\bar{\beta}$ such that for $\bar{\beta} < \beta$ and for $\underline{\kappa}_g < k < \overline{\kappa}_g$, the following is a subgame-perfect Nash equilibrium labelled Cooperation:

- *The storage units play cooperation.*
- *If one of the units deviates to competition when both units are either empty (state $(c_t^1 = 0, c_t^2 = 0)$) or full (state (k, k)), the other unit switches to competitive behavior forever as a punishment. That is, it starts buying (selling) every time when it is empty (full) under a favorable shock. A Nash Equilibrium of this subgame off the equilibrium path is either Competition, or FTL+Competition, or FTL, or the deviating unit quits the*

[18] It also implies the weaker condition $\frac{1-a}{1+a} < \frac{\delta\beta(1+\beta)}{4-\beta-\beta^2}$, which naturally implies (7) and guarantees a positive payoff to the second mover when cooperation is played. But it is not sufficient for cooperation to be an equilibrium.

market (after selling if full).

Here the threshold $\underline{\kappa}_g$ plays the same role as $\underline{\kappa}_b$ in Proposition 5: at that point, not only is cooperation more attractive than competition, it can be sustained as an equilibrium – with the threat of reverting to some non-cooperative equilibrium. Which non-cooperative equilibrium sustains the cooperative one depends on the capacity level. We show the payoff of the Cooperation equilibrium, and the range of capacity over which it can exist, in Figure 4, which also requires some comments. Characterising $\underline{\kappa}_g$ is a little more demanding than first appears.

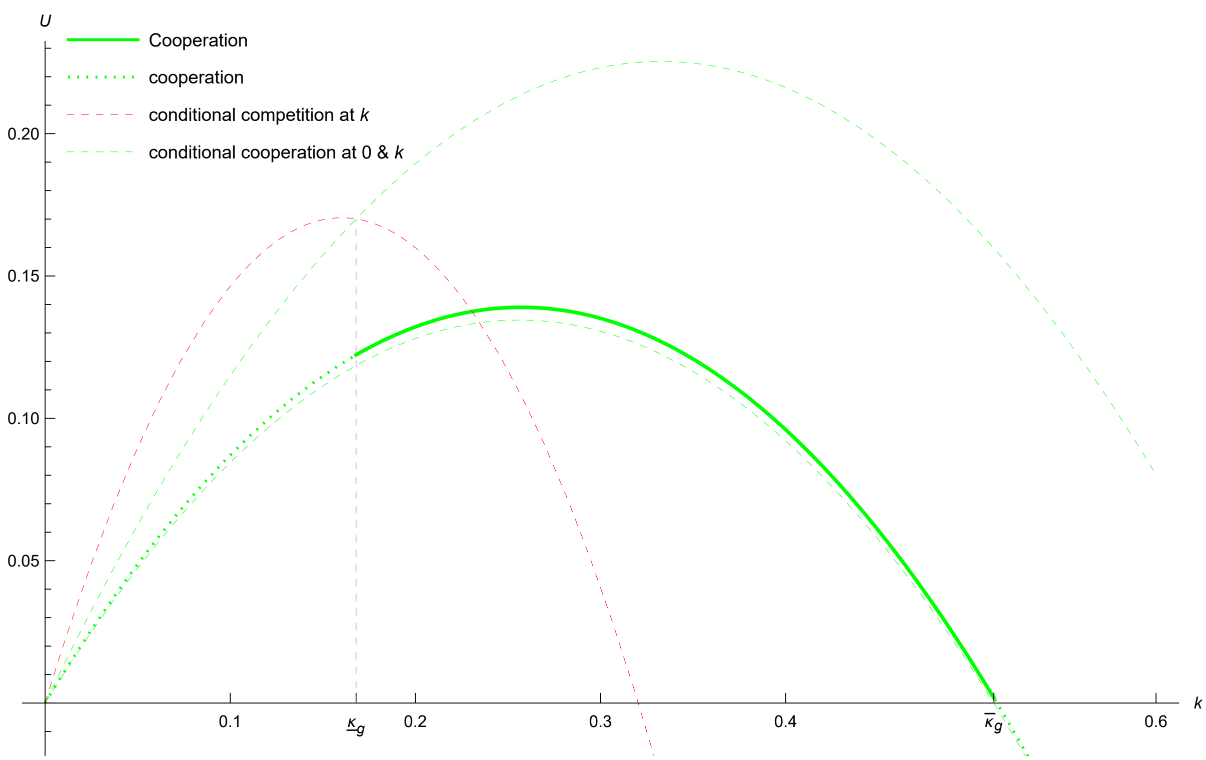

Figure 4: Collusion payoffs with the payoffs of possible deviations for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

As before, the subgame perfect criterion requires of us to consider deviations for all states $c_t = 0$ and $c_t = k$. To do so, one must compute the payoffs from cooperation for $c_t = 0$ and $c_t = k$, as well as the deviation payoff (to competition) for $c_t = k$. In Figure 4 these are labelled "conditional cooperation", with a clear ranking depending on c_t, and "conditional competition". The threshold $\underline{\kappa}_g$ is the level of capacity at which an operator with state $c_t = k$ is indifferent between cooperation and reverting to competition. Again these are *interim* payoffs, and they capture the relevant incentive here: what strategy should a full unit pursue at this stage of the game?[19] A full unit that plays Cooperation must wait, whereas it can sell immediately if competing. The threshold $\underline{\kappa}_g$ is then applied to the *ex ante* expected payoff from playing cooperation to determine the equilibrium. Likewise, the threshold $\overline{\kappa}_g$ is the level of capacity at which the *second-mover* with state $c_t = 0$ receives 0 from cooperation and so is better off doing something else. At that point, the empty unit that elects to cooperate must wait the longest: it must have a turn, charge, wait and then discharge. The green solid line shows the ex ante expected payoff to cooperation starting in state $c_t = 0$, and is an equilibrium over the range $[\underline{\kappa}_g, \overline{\kappa}_g]$. While not obvious from Figure 4, the ex ante equilibrium payoff is higher than the interim payoff with $c_t = 0$ even at $\overline{\kappa}_g$.

For capacities in excess of $\overline{\kappa}_g$, Cooperation can no longer be an equilibrium. Then players must revert to the asymmetric equilibria we identify in Section 3.2. Proposition 6 can be complemented with

Corollary 7. *There exist β^* such that for any $\beta > \beta^*$ Cooperation always exists for some values of a and δ; it does not exist otherwise.*

Corollary 7 identifies a necessary condition for the equilibrium Cooperation to exist. It can also be interpreted as a reaffirmation that Cooperation requires patience as players take turns to trade.

3.5 Payoff-maximising equilibria

Now we are in a position to collect our results and to characterise all payoff-maximising equilibria supported by a grim-trigger strategy. Recall the thresholds k_1 to k_3. As we know from Propositions 4 to 6, equilibria do not switch at these thresholds. However they do matter for incentives, and therefore ultimately to determine payoff-maximizing equilibria. We know from Lemma 2 that these thresholds are ordered; therefore competition, partial cooperation, and cooperation are always payoff-maximising behaviors (if positive) in this order.

[19] Comparing the payoffs to empty units is relevant for partial cooperation (Section 3.3).

Proposition 8. *If condition (7) holds, payoff-maximizing equilibria are characterized as follows:*[20]

1. *if Condition (8) holds and* $k_3 \leqslant \overline{\kappa}_g$,
   - *for* $k \in (0, \underline{\kappa}_b]$, *players engage in Competition;*
   - *for* $k \in [\underline{\kappa}_b, k_3]$, *players engage in Partial Cooperation;*
   - *for* $k \in [k_3, \overline{\kappa}_g]$, *full Cooperation prevails;*
2. *if Condition (8) fails, or* $k_3 > \overline{\kappa}_g$,
   - *for* $k \in (0, \underline{\kappa}_b]$, *players engage in Competition;*
   - *for* $k \in [\underline{\kappa}_b, \overline{\kappa}_b]$, *players engage in Partial Cooperation.*

This is laid out in Figure 5 (for $k_3 \leqslant \overline{\kappa}_g$) and Figure 6 (for $k_3 > \overline{\kappa}_g$). The solid lines depict equilibrium maximum payoffs that arise from the equilibria listed in Proposition 8. The dashed lines show payoffs arising from the same equilibria, but are payoff-dominated by another equilibrium. For example, in Figure 5, between $\underline{\kappa}_g$ and k_3, cooperation is an equilibrium behavior (the Cooperation equilibrium) but it is dominated by partial cooperation (the Partial Cooperation equilibrium) , which delivers higher payoffs. This is reversed for k larger than k_3.

[20] The relation between k_3 and $\overline{\kappa}_g$ is established for ease of exposition. Both are functions of the underlying parameters β, δ and a. So these conditions are equivalent to conditions on these underlying parameters, which we show in the Appendix but are too cumbersome to be helpful here.

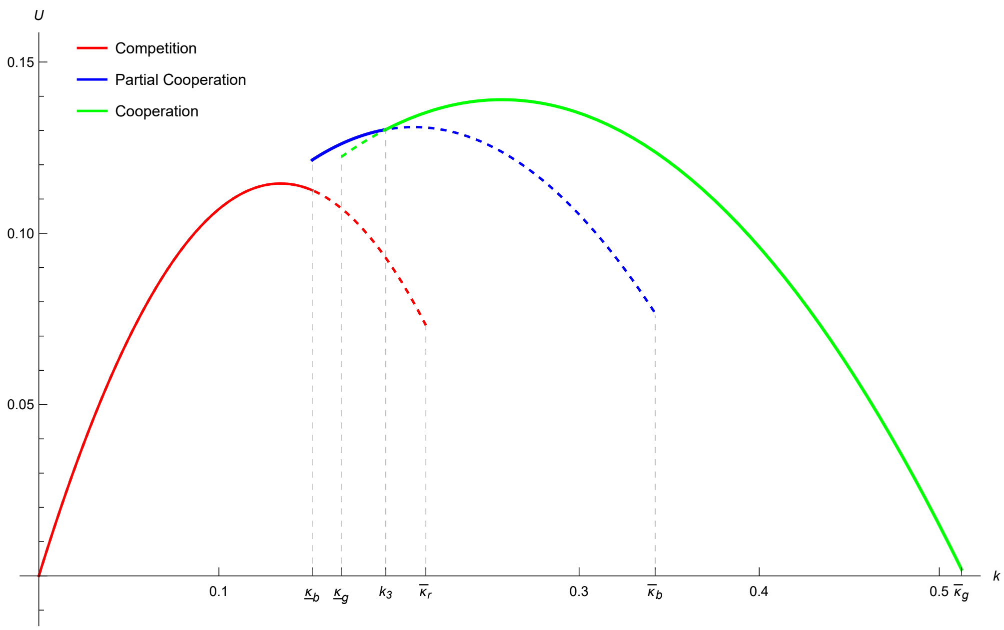

Figure 5: Equilibrium payoffs for different strategies for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$.

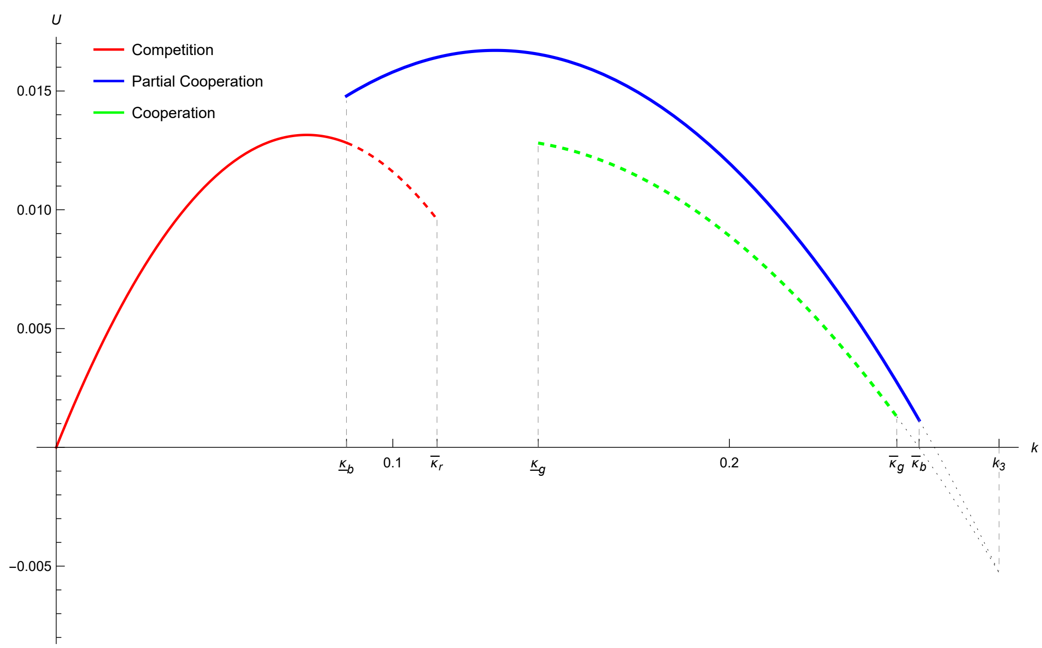

Figure 6: Equilibrium payoffs for different strategies for $n = 2$, $a = 0.6$, $\beta = 0.8$, $\delta = 0.85$.

There are important details that justify the length of Proposition 8 and the plethora of figures. The cooperative equilibria are both determined in relation to a version of a non-cooperative equilibrium (Competition, FTL+Competition or FTL). That is, for incentive purposes, these are the only equilibria that matter. There is no connection between Partial Cooperation and Cooperation as equilibria; in particular, one does not support the other. Rather, for a set of parameters (a, β, δ), they may just dominate one another in terms of payoffs only.

To illustrate, in both figures, at $\underline{\kappa}_b$, Partial Cooperation must deliver a discretely larger payoff than Competition otherwise the deviation is too tempting (equivalently, the benefit of Cooperation too small). This reflects the fact that Partial Cooperation must be robust to deviations at the interim stage – when one of the players is revealed to be the second mover and contemplates her options then (see Section 3.3). Likewise, at $\underline{\kappa}_g$, Cooperation must also deliver a discretely larger payoff than Competition: it must be robust to deviation at another interim stage (see Section 3.4). Further, in Figure 5, Cooperation not only exists as an equilibrium, it is also profit-maximising for a range of capacities. Figure 6 shows the complementary case. Cooperation is also an equilibrium for some capacities, but it is payoff-dominated by Partial Cooperation. The reason is that the threshold k_3, which is the indifference point between Partial Cooperation and Cooperation, falls to the right of $\overline{\kappa}_g$. At the point k_3, the payoffs to either behavior is negative.

Remark 9. *If in addition, Condition (8) fails, Cooperation does not even exist as an equilibrium because its payoff can never be positive.*

The contrast between Figure 1 and Remark 3 on the one hand, and Figures 5-6 and Proposition 8 is evident. Even under symmetric information, there exist frictions to cooperation because the incentives to the players differ in each state (c, ϵ).

Remark 10. *Instead of flipping a coin, one may consider allocating a token. Players flip a coin only once to decide who owns the token first. Whoever holds the token moves (buys or sells) first and passes the token to the other player. It's easy to show that the expected payoffs for cooperation and partial cooperation are the same, but the pivotal values of k change.*

4 Robustness: flexible actions and discounting

To this point, our results are derived using a rather inflexible set of actions: $b^i, s^i \in \{0, k\}$. It is natural then to wonder whether these results are not contrived and excessively specific to this rigid model. That is, do the incentives to collude remain so strong if players can soften competition unilaterally by altering the quantities they trade? And even if so, do the same equilibria still arise? Finally, we know behavior is sensitive to delay, which is why Partial Cooperation emerges as an equilibrium. So, how robust are these equilibria to changes in the discount factor?

4.1 More flexible actions

In brief, we are confident that our results are robust to a more flexible action space. In what follows, we carry out a similar exercise as in Section 3.1; that is, we compute the payoffs to the same behaviors when players can charge and discharge in two or three steps, rather than one. Employing two or three steps need not be optimal; indeed, in general, the optimal strategy ought to be a function of the state (here, $(\mathbf{c}, \epsilon)$) rather than be pre-specified. However one must accept that such a task lies beyond what is feasible today. To make progress we employ a similar approach as in our companion paper (Balakin and Roger (2023)) and rely on a *heuristic* to approximate an optimal strategy. This heuristic is to use constant quantities up to capacity.[21]

Except for a small subset of parameters, payoffs to these behaviors are ranked in the same order as for the single-step case. That is, for most parameter values, there exist similar indifference points to k_1, k_2, k_3. In addition, partial cooperation remains an attractive behavior for most parameter values. We do stop short from constructing equilibria for two and three steps, which is extremely tedious. However the same approach can be employed to verify (i) that cooperative equilibria exist and (ii) to construct them and (iii) find the payoff-maximising equilibria.

To see this, suppose now that the storage units can operate either in halves or thirds of their total capacity. As before let l denote the number of active units and m the number of steps (2 or 3) they use to charge or discharge their capacity k. Then we define charging costs (when purchasing energy) under the negative shock as $B_l(k/m)$ if l units buy k/m units of energy, and likewise the revenue a storage unit earns when selling k/m units of energy under

[21]Please see details of our companion paper to gain confidence in the robustness of this approach.

the positive shock as $A_l(k/m)$:

$$B_l\left(\frac{k}{m}\right) = \frac{1-a+lk/m}{n+1}\cdot\frac{k}{m}, \qquad A_l = \frac{1+a-l\delta k/m}{n+1}\cdot\frac{\delta k}{m}, \qquad l\in\{1,2\}, \quad m\in\{2,3\}.$$

As before, in both expressions, the right multiplier (k/m or $\delta k/m$) is just a quantity bought or sold. The left multiplier is the resulting Cournot price. Our n generators compete for the residual demand, which is now $1-a+lk/m$ or $1+a-l\delta k/m$ for negative and positive shocks, respectively. We can extend to $m=1$, in which case we are back in Section 3.1.

In light of the added flexibility provided by this heuristic, we need to amend the descriptions of the behaviors. By the same dominance argument, we dispense with two of the seven behaviors we start with.

1. *competition.* Storage units that are not full always buy energy together when they face negative shocks. They always sell energy together when they face positive shocks as long as they are not empty. Both storage units stay idle otherwise.

2. *partial cooperation.* If both storage units face negative shocks while not full, they flip a coin to decide who buys energy; the losing one remains idle. If both storage units face positive shocks while not empty, they sell simultaneously. If only one unit is full, the other one always buys under a negative shock. If only one unit is empty, the other one always sells under a positive shock. They stay idle otherwise.

3. *cooperation.* If both storage units face negative shocks while not full, they flip a coin to decide who buys; the losing one remains idle. If both storage units face positive shocks when not empty, they flip a coin again to decide who sells; the losing party remains idle. If only one unit is full, the other one always buys under a negative shock. If only one unit is empty, the other one always sells under a positive shock. They stay idle otherwise.

4. *follow the leader (ftl).* One of the units (the"leader") always buys (sells) first when it is not full (nonempty) under a negative (positive) shock. The second one (the "follower")

remains idle. Only if the leader is full, the follower who is not full can buy under a negative shock. Only if the leader is empty, the follower who is not empty can sell under a positive shock. They stay idle otherwise.

5. *ftl+competition.* The leader always buys first when both units are not full under a negative shock. The follower stays idle and can buy (under a negative shock) only when the leader is full. If both units face positive shocks while nonempty, they sell simultaneously. If the follower is empty and the leader is not, the latter sells alone under a positive shock. They stay idle otherwise.

The next two Propositions echo Proposition 1 and list the payoffs from the three behaviors of interest as functions of the capacity k.

Proposition 11. *Let $m = 2$. The payoffs from competition, partial cooperation and cooperation read:*

- *competition:*

$$U'_{com} = \frac{1}{2-\beta}\left(-B_2\left(\frac{k}{2}\right) + \frac{\beta}{(1-\beta)(2+\beta)}\left(A_2\left(\frac{k}{2}\right) - B_2\left(\frac{k}{2}\right)\right)\right).$$

- *partial cooperation:*

$$\begin{aligned} U'_{pc} &= \frac{1}{64-8\beta^2-12\beta^3+\beta^5}\Bigg[-4\left(4+2\beta+\beta^2\right)B_1\left(\frac{k}{2}\right) + \frac{\beta}{4(1-\beta)} \\ &\times \left(\left(4+\beta\right)\left(8-4\beta-\beta^3\right)\left(A_1\left(\frac{k}{2}\right) - B_1\left(\frac{k}{2}\right)\right) + 2\beta\left(8+4\beta+\beta^3\right)\left(A_2\left(\frac{k}{2}\right) - B_1\left(\frac{k}{2}\right)\right)\right)\Bigg] \end{aligned}$$

- *cooperation:*

$$U'_{col} = \frac{1}{4-2\beta-\beta^2}\left(-B_1\left(\frac{k}{2}\right) + \frac{\beta(2-\beta^2)}{(1-\beta)(4+2\beta-\beta^2)}\left(A_1\left(\frac{k}{2}\right) - B_1\left(\frac{k}{2}\right)\right)\right).$$

Proposition 12. *Let $m = 3$. The payoffs from competition, partial cooperation and cooperation read:*

- *competition:*

$$U''_{com} = \frac{1}{2(2-\beta^2)}\left(-(2+\beta)B_2\left(\frac{k}{3}\right) + \frac{\beta(4-\beta^2)}{4(1-\beta)}\left(A_2\left(\frac{k}{3}\right) - B_2\left(\frac{k}{3}\right)\right)\right).$$

- *partial cooperation:*

$$\begin{aligned} U''_{pc} &= \frac{1}{2H_1(\beta)}\left[-H_2(\beta)B_1\left(\frac{k}{3}\right)\right. \\ &+ \left.\frac{\beta}{1-\beta}\left(2H_3(\beta)\left(A_1\left(\frac{k}{3}\right) - B_1\left(\frac{k}{3}\right)\right) + \beta H_4(\beta)\left(A_2\left(\frac{k}{3}\right) - B_1\left(\frac{k}{3}\right)\right)\right)\right], \end{aligned}$$

where

$$H_1(\beta) = 2048 - 512\beta^2 - 896\beta^3 + 160\beta^5 + 40\beta^6 + 4\beta^8 - \beta^9,$$

$$H_2(\beta) = 1024 + 512\beta + 128\beta^2 - 288\beta^3 - 96\beta^4 + 24\beta^5 + 8\beta^6 + 2\beta^7 + \beta^8,$$

$$H_3(\beta) = 256 - 64\beta - 48\beta^2 - 96\beta^3 + 20\beta^4 + 10\beta^5 + \beta^6 - 2\beta^7,$$

$$H_4(\beta) = 256 + 64\beta - 80\beta^3 + 4\beta^5 + 8\beta^6 + \beta^7.$$

- *cooperation:*

$$U''_{col} = \frac{4-\beta^2}{2\left(8-4\beta-4\beta^2+\beta^3\right)}\left(-B_1\left(\frac{k}{3}\right) + \frac{\beta(4-3\beta^2)}{(1-\beta)\left(8+4\beta-4\beta^2-\beta^3\right)}\left(A_1\left(\frac{k}{3}\right) - B_1\left(\frac{k}{3}\right)\right)\right).$$

Next we depict these payoffs, sequentially in Figures 7 to 9, for $m = 1, 2, 3$ and for the same parameter values we use throughout Section 3. It is quite apparent that these payoffs are ordered in the same way throughout – for these parameters. We also point out that introducing flexibility in operations ($m = 2, 3$) has its own benefits, which we explain in detail in our companion paper.[22]

[22] Balakin and Roger (2023).

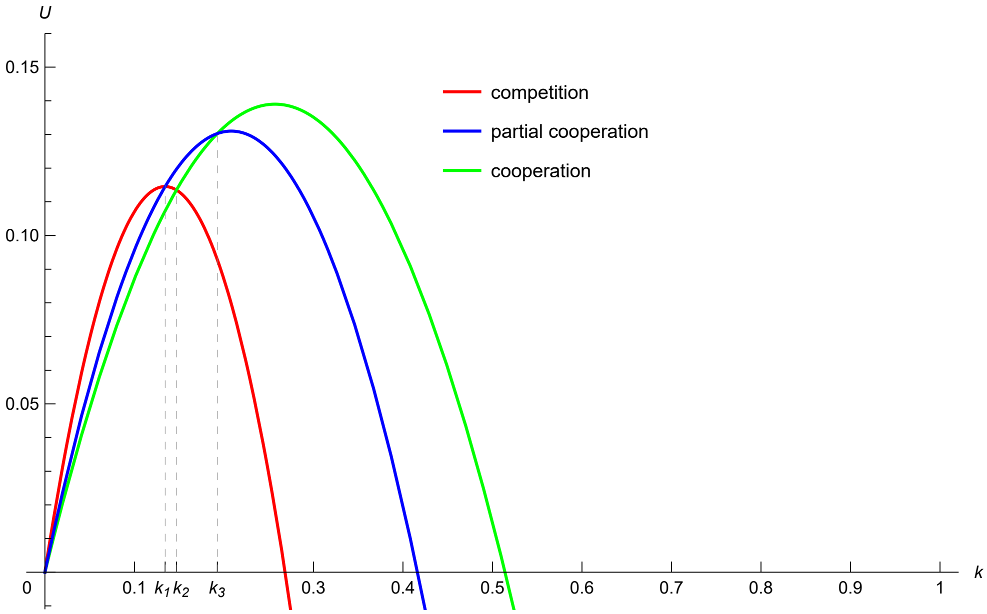

Figure 7: Payoffs for competition, pc, and cooperation for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$ and $m = 1$.

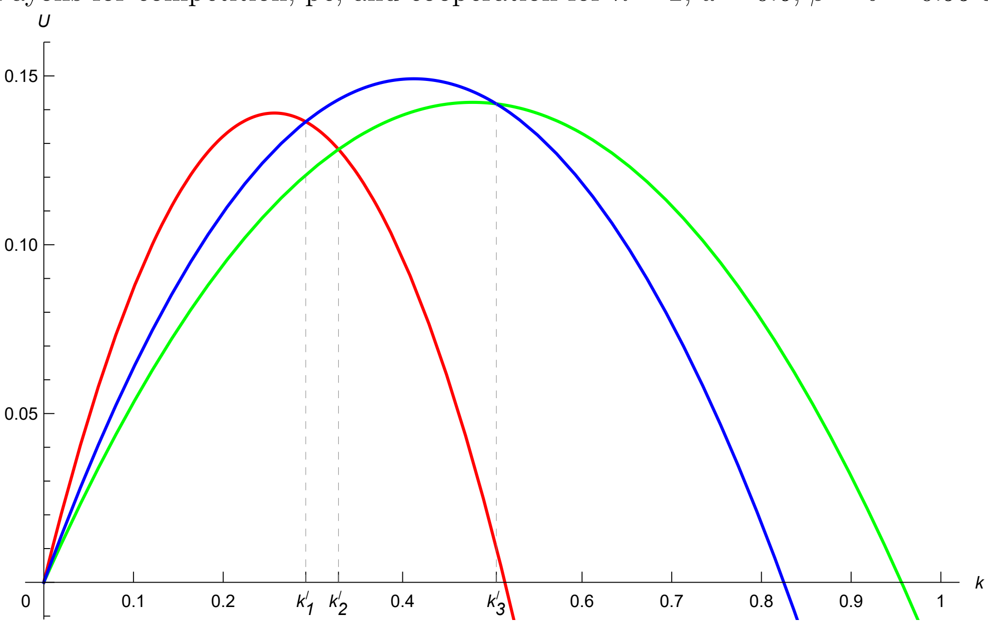

Figure 8: Payoffs for competition, pc, and cooperation for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$ and $m = 2$.

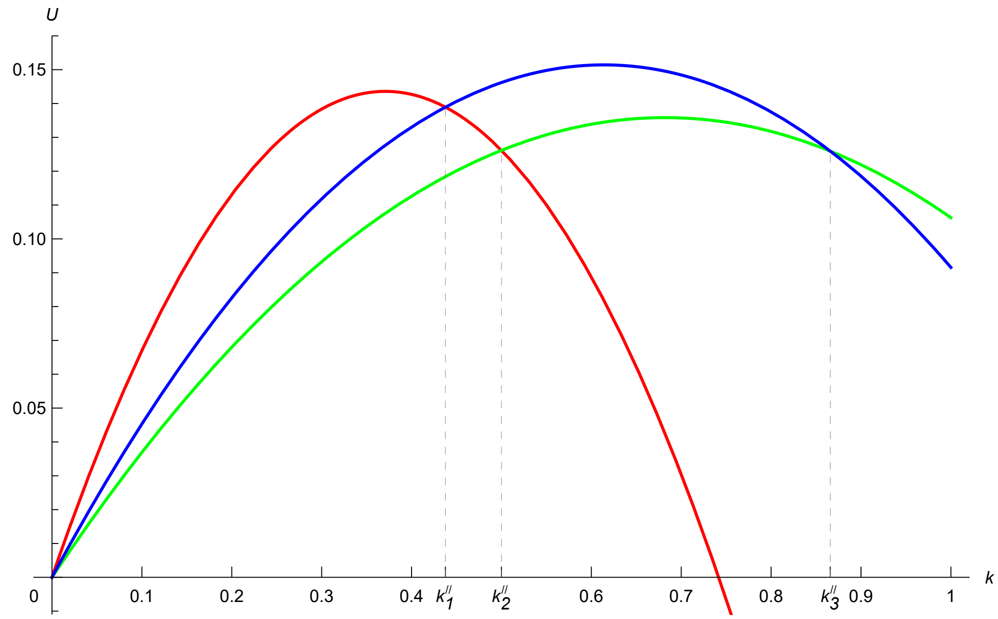

Figure 9: Payoffs for competition, pc, and cooperation for $n = 2$, $a = 0.6$, $\beta = \delta = 0.95$ and $m = 3$.

This series of pictures accords with the intuition that cooperation becomes less attractive with more flexible actions. We see in particular that partial cooperation delivers the highest payoffs when $m = 2$ and $m = 3$, however less so in the latter. As m increases, competing become relatively more attractive and cooperating can only payoff for large capacity. Let us try to make sense of this intuitive point, which actually has a quite subtle explanation.

If selling in multiple steps, for a fixed capacity each trade is small(er) and so does not erode the arbitrage spread as badly. Moreover, there are more opportunities to follow the behavior competition; there are m such relevant states. For example, for $m = 2$ *each* unit can buy in states $(0, 0)$ and $(k/2, k/2)$ and sell in states $(k/2, k/2)$ and (k, k).[23] The number of such opportunities, combined with the lesser price impact, render competition more attractive. On the other hand, any form of cooperation requires taking turns, precisely in these states where unilateral actions are available to *both*, which does include states $(0, k/2), (k/2, 0), (k, k/2)$ and $(k/2, k)$ as well. This is costly because of discounting. It is most costly under (full) cooperation, where units must forego trading opportunities buying and selling.[24] Hence for $m > 1$, units simultaneously face more opportunities to engage in competition (with a lesser

[23] The other states $(0, k/2), (k/2, 0)$ and $(k, k/2), (k/2, k)$ are never reached since players act symmetrically starting from $(0, 0)$.

[24] The only instance where cooperation is costless is when a unit must stay idle anyway, that is, in states $(k, 0)$ or $(0, k)$. But now of course the frequency of these events is lower.

price impact), and must forego unilateral action in more states if cooperating (for a lesser benefit).

None of this implies cooperation has no value; for large enough a capacity, it is still the more attractive behavior. But it is less compelling simply because flexibility in trade mutes the price impact. That is, fixing the discount factor, cooperation only becomes viable for capacities so large that even if selling in two or three steps, market power eats up the arbitrage spread. The indifference points increase with flexibility – see Figures 8 and 9. For large enough a capacity, the payoff from cooperation remains the most attractive, even as m increases.

Repeatedly we write "for almost all parameter values"; let's explain this now. The analogue of Lemma 2 fails to hold in general; that is, for some values of β, δ and a, partial cooperation is payoff-dominated by either competition or cooperation. On Figures 8 and 9 we see the indifference thresholds shift rightward. For some parameter values, for example $\beta = 0.998$, $\delta = 0.99$, $a = 0.8$, the thresholds k_2', k_2'' move right of k_3', k_3'', respectively; that is, partial cooperation is always payoff-dominated. The relevant parameter constellation requires very large values in all dimensions; indeed on Figures 8 and 9, where parameters take more intermediate values, the thresholds follow the same ordering as in Lemma 2. Hence, for some trading environments characterized by (β, δ, a), partial cooperation remains attractive. For others, in particular when β is very large and so corresponds to rapid trading, it does not. In Section B of the Appendix, we show complementary figures to illustrate the point.

4.2 Discounting

We know from the extensive literature on repeated and dynamic games that the existence of a cooperative equilibrium is sensitive to the discount factor. This is the dimension we want to explore here; to do so we fix $m = 2$. Again we stop short from exactly constructing equilibria; rather we compute the payoff functions from the three behaviors of interest as the discount factor varies from 0.6 to 0.999. The high values of the discount rate are meant to reflect the high frequency at which some commodities are traded – for example, electricity is traded every five minutes in Australia and every fifteen minutes in California. We display the graphs of these payoff functions in Figure 10.

First we observe that the ordering of the payoff functions does not change as the discount factor varies.[25] Second, cooperation becomes increasingly attractive as the discount factor increases, which is quite intuitive. More precisely, as capacity k increases, first partial cooperation and then cooperation deliver the highest payoffs. So, while flexibility in trade ($m = 2, 3, ...$) mitigates the incentives to collude, these incentives still remain and, as one may expect, become stronger the more patient the players are – or the higher the frequency of trade.[26] We conjecture with confidence that equilibria can be constructed as we proceed in Sections 3.2 to 3.4. This gives us comfort in the robustness of the analysis we carry out and the results we lay out in Section 3.

Figure 10 shows that Partial Cooperation is not just a "funny" equilibrium. For low-to-intermediate values of the discount factor, the behavior partial cooperation generates the highest payoffs and payoff-dominates cooperation for all relevant values of the capacity k (see the first three panels). This further illustrates the trade-off between restraining quantities and the cost of delay; for low discount factors, or equivalently, infrequent trading, delay is simply too costly.

In line with our earlier discussion, the last panel of Figure 10 ($\beta = 0.999$) suggests partial cooperation is almost payoff-dominated (it is not completely here). Bearing in mind that $m = 2$ (a small number), when the discount factor becomes (very) large, the cost of delay becomes negligible. This illustrates that the compromise that partial cooperation presents, becomes irrelevant.

[25]It takes large values of δ and a for this ordering to change; see Section B of the Appendix.

[26]In Sannikov and Skrzypacz (2007), cooperation breaks down as the frequency of play increases because of the inference problem; their game is one of incomplete information. There is no such inference to be made here.

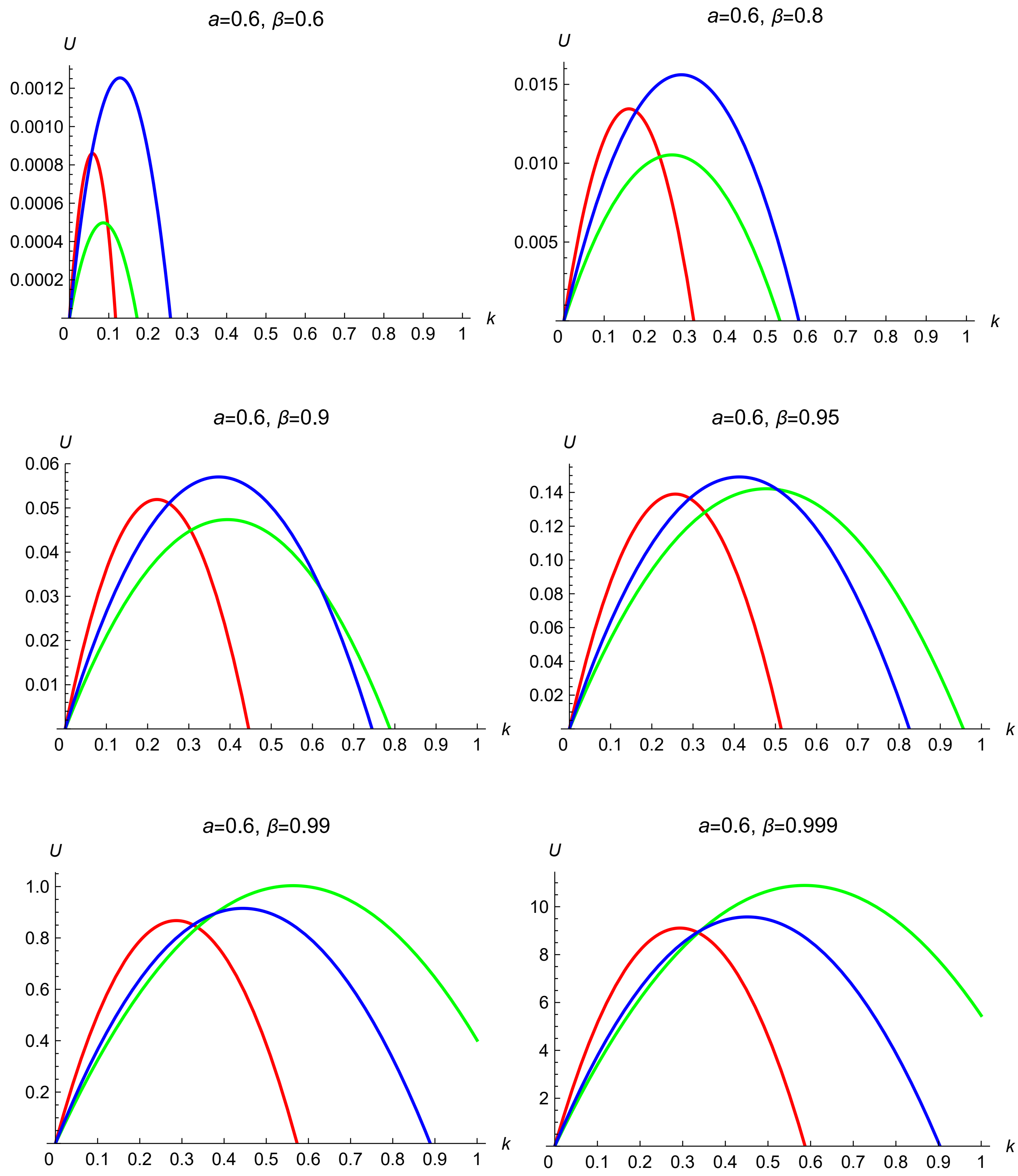

Figure 10: Payoffs for competition, partial cooperation, and cooperation for $n = 2,\ a = 0.6,\ \delta = 0.95$ and $m = 2$ as function of different values of β.

5 Implications

These findings have implications for competition policy. It is quite immediate that only small-capacity storage operators have any incentives to behave competitively. In electricity, a competition authority or a market operator (or both) may want to limit the size of any storage unit, especially for those like batteries, which offer little to no returns to scale. Where there are returns to scale, as for pumped hydroelectricity, it may be necessary to discipline these larger operators through a competitive fringe of small players.

Not only that, it is already not rare to see the same owner operate multiple units; for example, in Australia the French operator Neoen owns and operates three – and soon more – large units in the same wholesale electricity market. With batteries (in particular) it is trivial to coordinate the action of units that operate in the same portfolio. We suggest that this form of portfolio concentration presents real risks of anti-competitive behavior. First, they can easily coordinate on the units they own; second, as coordinated entity, they face exactly the incentives we study here.

In the same vein, some new business models emerge in the form of "Virtual Power Plants" (VPP), which propose to take control of multiple small storage units to trade in the wholesale market. Here too, a competition authority may need to cap the extent of the consolidation these businesses imply. And it may need to also want to prevent an entity from owning and operating multiple VPPs.

As argued before, our work applies beyond electricity to any storable commodity, such as those studied by Deaton and Laroque (1992) but also fuels and metals.[27] It may in fact also explain the emergence of bubbles in the trading of these commodities: speculators may hoard large stockpiles of commodities, which they then cannot get rid of without the price collapsing. In fact, this is what happened to Sumitomo's trader Yasuo Hamanaka in the mid-1990s on the London Metal Exchange.[28]

Finally, the restraining of quantities that we describe here – and which takes the form of

[27] Arguably, there is little arbitrage in the market for fresh tomatoes, for example; for such crop, simply set a low value to δ to render arbitrage uneconomical.

[28] Source: Reuters https://www.reuters.com/article/us-lme-warehousing-insight/insight-fixing-the-worlds-metals-warehousing-why-so-long-idUSBRE9AE06U20131115

cooperation between nominally competing storage units – has further consequences. Because purchase prices are kept low, they undervalue the traded commodity, which in turn leads to inefficiently low investment levels in the production of that commodity. In the context of electricity, this means insufficient investment in (renewable) generation capacity. Since we argue that storage is the bottleneck to the energy transition, this may call for some kind of intervention to alter the equilibrium behavior of storage operators.

6 Conclusion

Storage operators that have market power have very strong incentives to eschew competition because the arbitrage spread they need to earn squeezes rapidly to oblivion from their selling and their *buying*. We show these incentives continue to exist even when storage units can unilaterally decrease their traded quantities.

Cooperation is a standard avenue competitors can use to tame the impact of their own market power. Here it can be implemented in multiple ways because the decision whether to collude has to be made for the actions of buying, selling or both. We uncover a new class of equilibrium that we call Partial Cooperation that reflect this feature. Partial Cooperation is the embodiment of a compromise between restraining quantities (when buying) and facing the cost of delay from discounting, which leads to selling simultaneously. Because of the dynamic nature of the game and of these multiple actions (selling and buying), the construction of equilibria can be intricate at times. The reason is that the exact incentives depend on both the state of charge of the units and of their current action.

We also find that head-on, repeated competition is not always an equilibrium – whereas it is in the standard Cournot game, for example. Again, the reason is that with large enough capacities, head-on competition induces the arbitrage spread to vanish. Instead, asymmetric equilibria emerge, in which one of the players accepts to be a follower and to play only when the circumstances allow it, rather than seeking to compete.

Finally we draw some implications of these results, both in terms of policy and of the development of an industry, in which storage plays an important role. When storage acts non-competitively, it depresses the purchase price of the underlying commodity and therefore its value to potential investors.

A Complement to Proposition 8

In Proposition 8 we restrict attention to large enough a discount factor β, as is standard in the literature on repeated games and stochastic games; it also assists in the exposition. Other (payoff-maximizing) equilibria exist for smaller β, such that $0.631 < \beta < 0.81$, and for some values of a and δ, even if $k_3 < \underline{\kappa}_g$. Here we present these complementary cases in pictures.

Two options emerge: $\overline{\kappa}_b \geqslant \underline{\kappa}_g$ and $\overline{\kappa}_b < \underline{\kappa}_g$, described by Figures 11 and 12, respectively. In the first case, we observe a new jump from the blue line to the green line at point $\underline{\kappa}_g$. In the second case, where β is smaller, there is a gap $(\overline{\kappa}_b, \underline{\kappa}_g)$, where none of those two equilibria exist.

These two additional cases exhaust all possibilities. We present them here for completeness and regards them more as curiosities rather than central to our analysis.

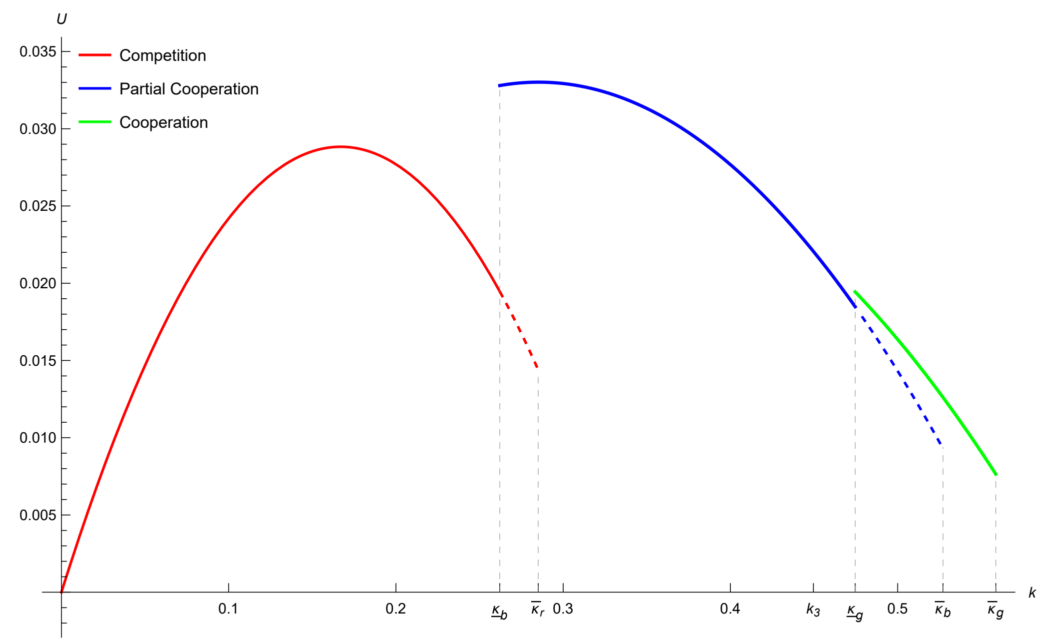

Figure 11: Equilibrium payoffs for different strategies for $n = 2$, $a = 0.99$, $\beta = 0.68$, $\delta = 0.99$.

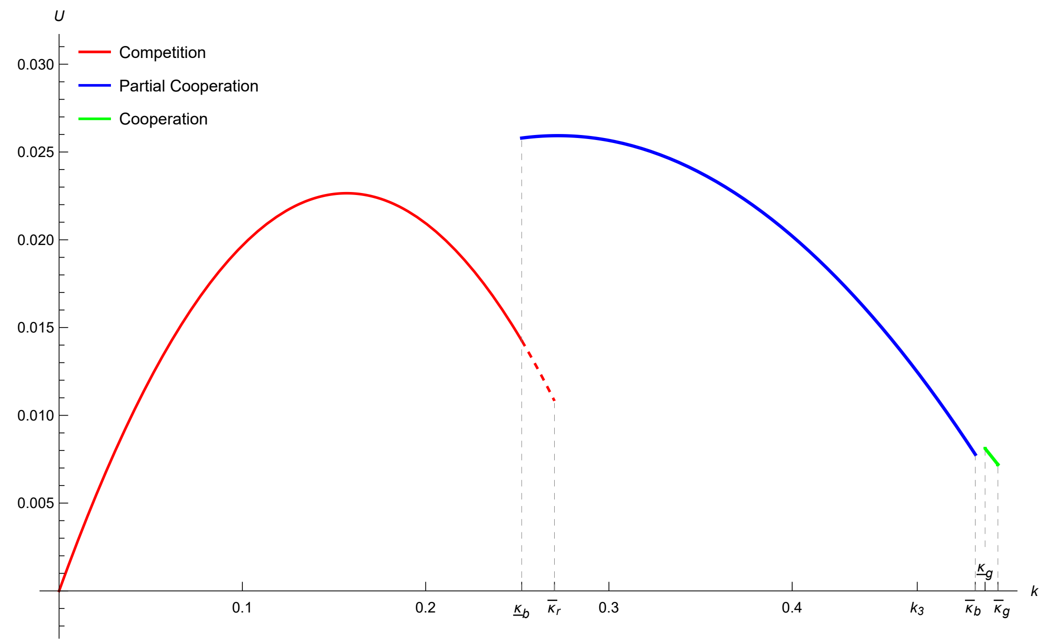

Figure 12: Equilibrium payoffs for different strategies for $n = 2$, $a = 0.99$, $\beta = 0.64$, $\delta = 0.99$.

B Complement to Section 4.1

Here we illustrate the point that partial cooperation becomes payoff dominated for large values of *all* parameters (β, δ, a). The most sensitive parameter is the discount factor β. We compute and graph payoffs for the case where partial cooperation still has a role to play ($\beta = 0.996$, $\delta = 0.99$, $a = 0.9$ for $m = 2$ and $\beta = 0.99$, $\delta = 0.99$, $a = 0.9$ for $m = 3$) and where it is dominated ($\beta = 0.998$, $\delta = 0.99$, $a = 0.9$ for $m = 2$ and $\beta = 0.992$, $\delta = 0.99$, $a = 0.9$ for $m = 3$). We "zoom in" to show how minor an event it is. The axes and colors are the same as for all other figures. In Figures 13 and 14 the range of relevant capacities is $[0.5, 0.53]$. In Figure 13, even for a large value of β, the ranking of the indifference thresholds k'_1, k'_2, k'_3 is the same as in Lemma 2.

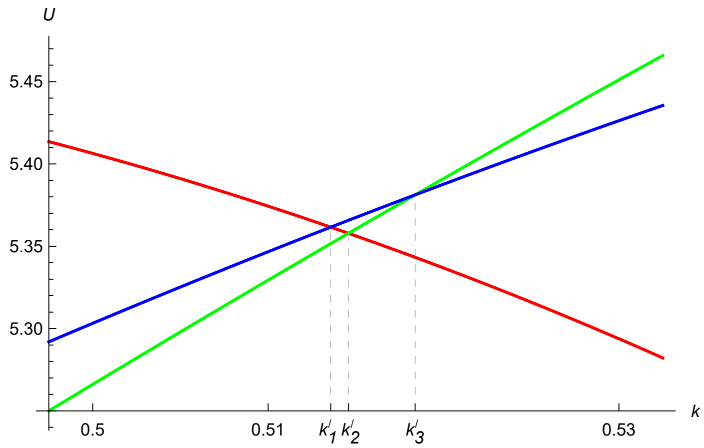

Figure 13: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.996$, $\delta = 0.99$ and $m = 2$.

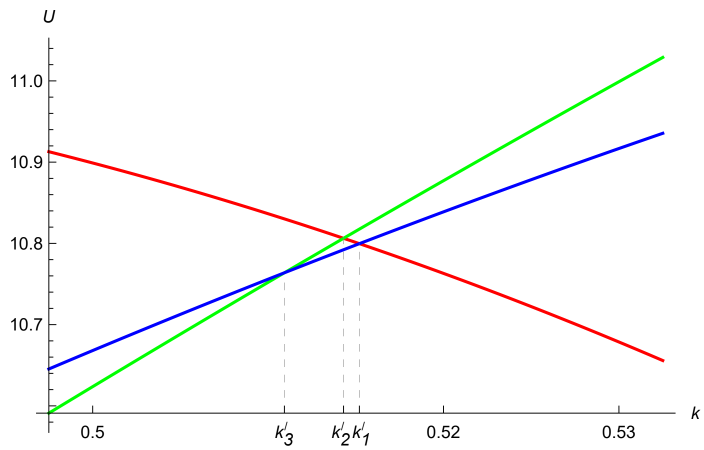

Figure 14: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.998$, $\delta = 0.99$ and $m = 2$.

In Figure 14, for a slightly larger value of β, the ranking of these indifference thresholds is reversed. In Figures 15 and 16 the range of relevant capacities is higher: $[0.795, 0.82]$.

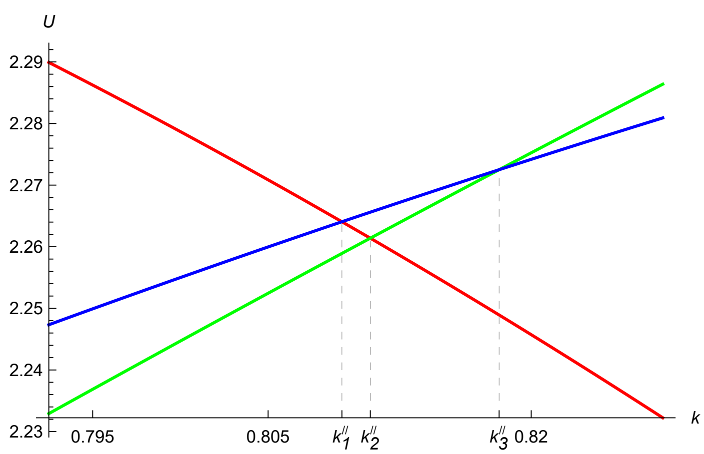

Figure 15: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.99$, $\delta = 0.99$ and $m = 3$.

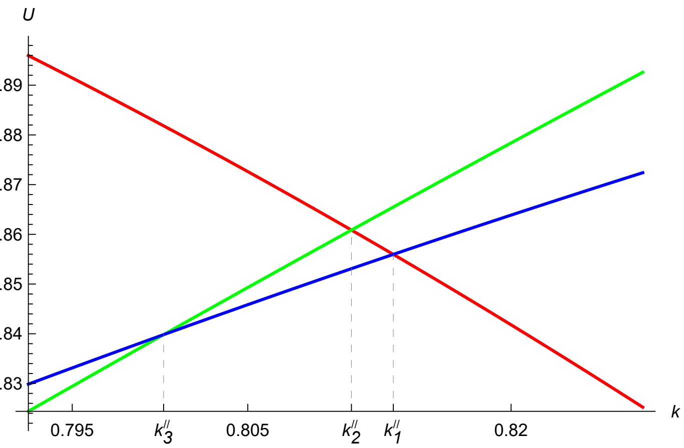

Figure 16: Payoffs for competition, partial cooperation, and cooperation for $n = 2$, $a = 0.9$, $\beta = 0.992$, $\delta = 0.99$ and $m = 3$.

We observe the same ranking and its reversal for lower values of the discount factor β (however still large). This also confirms the idea that partial cooperation is a behavior that is a compromise between quantity restriction and time delay that emerges most naturally when actions are inflexible.

C Proofs–for online publication

Proof of Proposition 1. We start in the order of the proof with the payoff formula for competition first. Since storage units do everything symmetrically, there are only two states of charge here, $(0, 0)$ and (k, k). The system of equations (6) takes the following form:

$$\begin{cases} V(0,0) = \frac{\beta}{2} V(0,0) + \frac{1}{2}\left(-B_2 + \beta V(k,k)\right), \\ V(k,k) = \frac{\beta}{2} V(k,k) + \frac{1}{2}\left(A_2 + \beta V(0,0)\right). \end{cases}$$

Indeed, if both units are empty, they either remain empty in the case of a positive shock (with probability 1/2) or compete while purchasing energy (and spending B_2) under a negative shock. In the latter case, the new state of charge is (k, k). Likewise, if both units are full, they either remain full in the case of a negative shock (with probability 1/2) or compete while selling energy (and gaining A_2) under a positive shock. Then they return to state $(0, 0)$.

Using the notation from equation (6),

$$\mathbf{P} = \begin{pmatrix} -B_2 \\ A_2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}.$$

Note that $\mathbf{Q}^2 = \mathbf{Q}$. For $\beta < 1$, we can find that

$$\mathbf{V} = \mathbf{P} + \sum_{i=1}^{t} \beta^i \mathbf{Q}^i \cdot \mathbf{P} + \beta^{t+1}\mathbf{Q}^{t+1} \cdot \mathbf{V} =$$

$$= \mathbf{P} + \frac{\beta(1-\beta^t)}{1-\beta} \cdot \mathbf{Q} \cdot \mathbf{P} + \beta^{t+1} \cdot \mathbf{Q} \cdot \mathbf{V} \xrightarrow[t\to\infty]{} \mathbf{P} + \frac{\beta}{1-\beta} \cdot \mathbf{Q} \cdot \mathbf{P} = \frac{1}{4(1-\beta)} \begin{pmatrix} \beta A_2 - (2-\beta)B_2 \\ (2-\beta)A_2 - \beta B_2 \end{pmatrix}.$$

The upper term $V(0,0)$ is exactly U_{com}.

Note now that the leader's payoff for ftl is the same except for substituting A_2 and B_2 for A_1 and B_1, respectively. It is implied by the fact that the leader acts exactly the same way as the units do under competition (always buying when empty under a negative shock and always selling when full under a positive shock), but it doesn't face any competition from the follower.

Next we turn to partial cooperation. The system of equations (6) takes the following form:

$$\begin{cases} V(0,0) = \frac{\beta}{2}V(0,0) + \frac{1}{4}\left(-B_1 + \beta V(k,0)\right) + \frac{1}{4}\beta V(0,k), \\ V(0,k) = \frac{\beta}{2}V(0,0) + \frac{1}{2}\left(-B_1 + \beta V(k,k)\right), \\ V(k,0) = \frac{\beta}{2}V(k,k) + \frac{1}{2}\left(A_1 + \beta V(0,0)\right), \\ V(k,k) = \frac{\beta}{2}V(k,k) + \frac{1}{2}\left(A_2 + \beta V(0,0)\right). \end{cases}$$

Indeed, if both units are empty, with probability 1/2 they experience a negative shock and remain empty (with discounting). However, if the shock is positive (probability 1/2 again), they flip the coin. With resulting probability 1/4 the first unit buys energy alone, pays B_1, and we end up with state $(k,0)$ where this unit is full and the other one is still empty. With the same resulting probability 1/4 the other unit purchases up to its full capacity, while the first unit stays idle. In this case, we move to state $(0, k)$.

If the first unit is empty and the second one is full (state $(0, k)$), the first unit either stays idle in the case of a positive shock (and we turn to $(0, 0)$ afterwards because the second unit sells) or buys energy alone paying B_1 in the case of a negative shock (and the new state will be (k, k)). Likewise, in the case of state $(k, 0)$ the first unit stays either idle in the case of negative shock (and the other unit buys shifting the state of charge to (k, k)) or sells energy alone gaining A_1 in the case of positive shock (the new state is $(0, 0)$).

Finally, when both units are full, they stay idle with probability 1/2 in the case of negative shock and compete otherwise. In this case both units get A_2 for selling their energy and become empty (state $(0, 0)$).

Again using the notation from equation (6),

$$\mathbf{P} = \begin{pmatrix} -B_1/4 \\ -B_1/2 \\ A_1/2 \\ A_2/2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}.$$

Note that for any $n > 2$

$$\mathbf{Q}^n = \mathbf{Q}^2 = \begin{pmatrix} 1/2 & 1/8 & 1/8 & 1/4 \\ 1/2 & 1/8 & 1/8 & 1/4 \\ 1/2 & 1/8 & 1/8 & 1/4 \\ 1/2 & 1/8 & 1/8 & 1/4 \end{pmatrix}.$$

Then we can find $\mathbf{V}(\mathbf{c^1}, \mathbf{c^2}) = \mathbf{V}$ for any $\beta < 1$.

$$\mathbf{V} = \mathbf{P} + \beta \cdot \mathbf{Q} \cdot \mathbf{P} + \sum_{i=2}^{t} \beta^i \cdot \mathbf{Q}^i \cdot \mathbf{P} + \beta^{t+1} \cdot \mathbf{Q}^{t+1} \cdot \mathbf{V} =$$

$$= \mathbf{P} + \beta \cdot \mathbf{Q} \cdot \mathbf{P} + \frac{\beta^2(1-\beta^{t-1})}{1-\beta} \cdot \mathbf{Q}^2 \cdot \mathbf{P} + \beta^{t+1} \cdot \mathbf{Q}^2 \cdot \mathbf{V} \xrightarrow[t\to\infty]{} \mathbf{P} + \beta \cdot \mathbf{Q} \cdot \mathbf{P} + \frac{\beta^2}{1-\beta} \cdot \mathbf{Q}^2 \cdot \mathbf{P} =$$

$$= \frac{1}{16(1-\beta)} \begin{pmatrix} 2\beta^2 A_2 + \beta(2-\beta)A_1 - (4-\beta^2)B_1 \\ 2\beta(2-\beta)A_2 + \beta^2 A_1 - (2-\beta)(4-\beta)B_1 \\ 2\beta(2-\beta)A_2 + (8-8\beta+\beta^2)A_1 - \beta(2+\beta)B_1 \\ 2(4-2\beta-\beta^2)A_2 + \beta^2 A_1 - \beta(2+\beta)B_1 \end{pmatrix}.$$

The uppermost term is exactly $V(0,0) = U_{pc}$.

The leader's and the follower's payoffs $\bar{U}_{fc}$ and U_{fc} for ftl+competition can be obtained the same way as U_{pc}. Indeed, for the leader we have:

$$\mathbf{P} = \begin{pmatrix} -B_1/2 \\ -B_1/2 \\ A_1/2 \\ A_2/2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix},$$

$$\mathbf{Q}^n = \mathbf{Q}^2 = \begin{pmatrix} 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \end{pmatrix}, \qquad n > 2.$$

Then

$$V(0,0) = \frac{1}{8(1-\beta)} \left(\beta^2 A_2 + (2-\beta)(-2B_1 + \beta A_1)\right),$$

which is exactly $\bar{U}_{fc}$.

For the follower in ftl+competition we have:

$$\mathbf{P} = \begin{pmatrix} 0 \\ -B_1/2 \\ A_1/2 \\ A_2/2 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix},$$

$$\mathbf{Q}^n = \mathbf{Q}^2 = \begin{pmatrix} 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 & 1/4 \end{pmatrix}, \qquad n > 2.$$

Then

$$V(0,0) = \frac{\beta}{8(1-\beta)} \left(\beta A_2 - (2-\beta) B_1\right),$$

which is exactly U_{fc}.

Now let's obtain the payoff formula for cooperation. The vector $\mathbf{P}$ and matrix $\mathbf{Q}$ from (6) take the following form:

$$\mathbf{P} = \begin{pmatrix} -B_1/4 \\ -B_1/2 \\ A_1/2 \\ A_1/4 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 1/4 & 1/4 & 1/2 \end{pmatrix}.$$

To calculate power t of matrix $\mathbf{Q}$, we find the Jordan decomposition $\mathbf{Q} = \mathbf{T} \cdot \mathbf{J} \cdot \mathbf{T}^{-1}$ of $\mathbf{Q}$. Here,

$$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & -2 & 0 & -1 \\ 1 & -2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix},$$

so $\mathbf{Q}^t = \mathbf{T} \cdot \mathbf{J}^t \cdot \mathbf{T}^{-1}$. For $\beta < 1$,

$$\mathbf{V} = \mathbf{P} + \sum_{i=1}^{\infty} \beta^i \mathbf{Q}^i \cdot \mathbf{P} = \mathbf{P} + \mathbf{T} \cdot \begin{pmatrix} \frac{\beta}{1-\beta} & 0 & 0 & 0 \\ 0 & -\frac{\beta}{2+\beta} & 0 & 0 \\ 0 & 0 & \frac{\beta}{2-\beta} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \mathbf{T}^{-1} \cdot \mathbf{P} =$$

$$= \frac{1}{4(1-\beta)(2-\beta)(2+\beta)} \begin{pmatrix} 2\left(\beta A_1 - (2-\beta^2)B_1\right) \\ (2-\beta)\left(\beta(1+\beta)A_1 - (4-\beta-\beta^2)B_1\right) \\ (2-\beta)\left((4-\beta-\beta^2)A_1 - \beta(1+\beta)B_1\right) \\ 2\left((2-\beta^2)A_1 - \beta B_1\right) \end{pmatrix}.$$

The uppermost term is exactly $V(0,0) = U_{col}$.

Finally, the follower's payoff U_{ftl} for ftl may be obtained the same way as U_{col}. Vector $\mathbf{P}$ and matrix $\mathbf{Q}$ from (6) take the following form:

$$\mathbf{P} = \begin{pmatrix} 0 \\ -B_1/2 \\ A_1/2 \\ 0 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}.$$

Then matrices $\mathbf{J}$ and $\mathbf{T}$ are the following:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -2 & 0 & 1 \\ 1 & -2 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

For $\beta < 1$, we have

$$V(0,0) = \frac{\beta}{4(1-\beta)(2-\beta)(2+\beta)} \left(\beta^2 A_1 - (4-2\beta-\beta^2)B_1\right),$$

which is exactly U_{ftl}. □

Proof of Lemma 2. The real numbers k_1, k_2, k_3 are nonzero roots of equations $U_{com}(k) = U_{pc}(k)$, $U_{com}(k) = U_{col}(k)$, and $U_{col}(k) = U_{pc}(k)$, respectively. Solving them, we obtain:

$$k_1 = \frac{-(2-\beta)(1-a)+\beta(1+a)\delta}{6-\beta+3\beta\delta^2} > 0, \qquad k_2 = \frac{-(4-4\beta+\beta^3)(1-a)+\beta(2-\beta^2)(1+a)\delta}{2\left(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\right)},$$
$$k_3 = \frac{-\beta^2(1-a)+(4-2\beta-\beta^2)(1+a)\delta}{\beta^2+(12-2\beta-3\beta^2)\delta^2}.$$

Then, using (7), we get:

$$\begin{aligned}
k_2 - k_1 &= \frac{\beta^2\Big((-\beta^2+(6-4\beta-\beta^2)\delta^2)(1-a)+(6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\delta\Big)}{2\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)\big(6-\beta+3\beta\delta^2\big)} > \\
&> \frac{\beta^2\left((-\beta^2+(6-4\beta-\beta^2)\delta^2)(1-a)+(6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}\right)}{2\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)\big(6-\beta+3\beta\delta^2\big)} = \\
&= \frac{\beta(1-\beta)(1-a)}{6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2} > 0,
\end{aligned}$$

$$\begin{aligned}
k_3 - k_2 &= \frac{(2+\beta)(2-\beta)^2\Big((-\beta^2+(6-4\beta-\beta^2)\delta^2)(1-a)+(6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\delta\Big)}{2\big(\beta^2+(12-2\beta-3\beta^2)\delta^2\big)\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)} > \\
&> \frac{(2+\beta)(2-\beta)^2\left((-\beta^2+(6-4\beta-\beta^2)\delta^2)(1-a)+(6-4\beta-\beta^2-\beta^2\delta^2)(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}\right)}{2\big(\beta^2+(12-2\beta-3\beta^2)\delta^2\big)\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)} = \\
&= \frac{(2+\beta)(1-\beta)(2-\beta)^2\big(6-\beta+3\beta\delta^2\big)(1-a)}{\beta\big(\beta^2+(12-2\beta-3\beta^2)\delta^2\big)\big(6-4\beta-\beta^2+\beta^3+\beta(3-\beta^2)\delta^2\big)} > 0,
\end{aligned}$$

□

Proof of Proposition 4. Condition (7) guarantees that the competitive payoff is positive for some $k > 0$. Indeed, the second nonzero root k_r of the equation $U_{com} = 0$ is equal to

$$k_r = \frac{-(2-\beta)(1-a)+\beta(1+a)\delta}{2\left(2-\beta+\beta\delta^2\right)},$$

which is positive if and only if (7) holds.

Since payoffs are expressed as a function of the capacity k, we construct equilibria for ranges of the value of that capacity. So whether a behavior is an equilibrium is determined in terms of that capacity. Deviations from competition may be only in the form of not competing but allowing the opponent to buy or sell first alone. Consider different deviations:

- Both players are empty (so $c_t^1 = c_t^2 = 0$) and ready to buy under the negative shock, but one player deviates by **not buying** competitively just for **one period**. They continue competing after that. This deviation is profitable when

$$k > x_b = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{4 - \beta + 2\beta\delta^2}.$$

  Indeed, the payoff U_0 of each player if no deviation is observed (conditional on a negative shock and on both storage units being empty) is equal to

$$U_0 = -B_2 + \beta V(k,k) = -B_2 + \frac{\beta}{2}A_2 + \frac{\beta^2}{4(1-\beta)}(A_2 - B_2),$$

  The payoff U_d of a deviating player is

$$U_d = \beta\left(\frac{1}{2}(-B_1 + \beta V(k,k)) + \frac{1}{2}\beta V(0,0)\right) = \beta\left(-\frac{B_1}{2} + \frac{\beta}{4(1-\beta)}(A_2 - B_2)\right).$$

  Here, $V(k,k)$ and $V(0,0) = U_{com}$ are continuation values for competition when both players are full (state $(c_t^1 = k, c_t^2 = k)$) and empty (state $(c_t^1 = 0, c_t^2 = 0)$), respectively (see Proof of Prop. 1). The deviation is profitable if $U_0 < U_d$, which is equivalent to $k > x_b$.

The threshold x_b stays constant in the case of deviation for not buying competitively for two, three, four, etc. periods of time up to infinity. The latter means that the deviating player completely switches to the behavior ftl+competition. Thus, x_b is the point where the red competition curve intersects with the orange ftl+competition line (see Fig. 2).

- Both players are full ($c_t^1 = k, c_t^2 = k$) and ready to sell under the positive shock, but one player deviates by **not selling** competitively just for **one period**. They continue competing after that. This deviation is profitable when

$$k > x_s = \frac{-\beta(1-a) + (2-\beta)(1+a)\delta}{2\beta + (4-\beta)\delta^2}.$$

Indeed, the payoff U_k of each player if no deviation is observed (conditional on a positive shock and on both storage units being full) is equal to

$$U_k = A_2 + \beta V(0,0) = A_2 - \frac{\beta}{2}B_2 + \frac{\beta^2}{4(1-\beta)}(A_2 - B_2),$$

The payoff $U_{d'}$ of a deviating player is

$$U_{d'} = \beta\left(\frac{1}{2}\left(A_1 + \beta V(0,0)\right) + \frac{1}{2}\beta V(k,k)\right) = \beta\left(\frac{A_1}{2} + \frac{\beta}{4(1-\beta)}\left(A_2 - B_2\right)\right).$$

The deviation is profitable if $U_k < U_{d'}$, which is equivalent to $k > x_s$.

Again, the threshold x_s remains constant for the *same* deviation for two, three, four, etc. periods of time up to infinity. The latter means that the deviating player completely switches to competition+ftl. Now we prove that $x_b < x_s$. Indeed, using (7), we get

$$\begin{aligned} x_s - x_b &= \frac{\left(-\beta^2 + (8-6\beta-\beta^2)\delta^2\right)(1-a) + \left(8-6\beta-\beta^2-\beta^2\delta^2\right)(1+a)\delta}{\left(2\beta+(4-\beta)\delta^2\right)\left(4-\beta+2\beta\delta^2\right)} \\ &> \frac{\left(-\beta^2 + (8-6\beta-\beta^2)\delta^2\right)(1-a) + \left(8-6\beta-\beta^2-\beta^2\delta^2\right)(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}}{\left(2\beta+(4-\beta)\delta^2\right)\left(4-\beta+2\beta\delta^2\right)} \\ &= \frac{4(1-\beta)(1-a)}{\beta\left(2\beta+(4-\beta)\delta^2\right)} > 0. \end{aligned}$$

There are potentially many more deviations, but now that x_b does not change by adding "buying" deviations, the only way the pivotal value of k can change is by combining instances of buying and selling. Likewise with x_s and selling.

For k in the interval $[x_b, x_s]$, all deviations ("letting go") when buying are profitable, but not when selling. Starting from x_b, adding "selling" deviations moves the pivotal point to the right towards x_s – that is, the pivotal $k > x_b$ so a deviation is more demanding in that it requires a larger capacity to be profitable. Conversely, starting from x_s, adding "buying" deviations moves the pivotal point to the left towards x_b – that is, the pivotal $k < x_s$. For example, consider pivotal values for the following deviations:

- x_{bs} – deviation to one instance of buying and maximum one instance of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock;
- x_{sb} – the converse, however starting from $(c_t^1 = k, c_t^2 = k)$ and facing the positive shock;
- $x_{b\infty s\infty}$ – deviation forever starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock (switch to ftl strategy);
- $x_{s\infty b\infty}$ – deviation forever starting from $(c_t^1 = k, c_t^2 = k)$ and facing the positive shock;
- x_{b2s} – deviation to instances of buying and at most one of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock;
- x_{bs2} – deviation to one instance of buying and at most two of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock;
- x_{b2s2} – deviation to at most two instances of buying and at most two of selling starting from $(c_t^1 = 0, c_t^2 = 0)$ and facing the negative shock.

Then we can rank the respective indifference thresholds in the following order:

$$x_b \leqslant x_{b2s} \leqslant x_{bs} \leqslant x_{b2s2} \leqslant x_{bs2} \leqslant x_{b\infty s\infty} \leqslant x_{s\infty b\infty} \leqslant x_{sb} \leqslant x_s.$$

We remark that

- $x_{b\infty s\infty} = x_{bs\infty} = x_{b2s\infty}$ with the same idea as for $x_b = x_{b2} = x_{b3} = \ldots = x_{b\infty}$. That is, fixing one behavior renders the thresholds constant.
- *All* pivotal values for all the deviations that start from $(c_t^1 = 0, c_t^2 = 0)$ are lower than the pivotal values for all the deviations that start from $(c_t^1 = k, c_t^2 = k)$. Moreover, these intervals do not intersect except for the case $\beta = 1$. Then $x_{b\infty s\infty} = x_{s\infty b\infty}$, otherwise $x_{b\infty s\infty} < x_{s\infty b\infty}$.

- The effect of additional instances of buying and selling cancel each other if and only if $\beta = 1$ and $\delta = 1$. In this case, not only $x_{bs} = x_{b2s2}$ but also $x_b = x_s$; the one pivotal value is the same for all candidate deviations.

Finally, we prove that $x_b \leqslant k_r$ to make sure that at least (some of) our candidate deviations are not trivial.

$$k_r - x_b = \frac{\beta\left(-(2-\beta)(1-a) + \beta(1+a)\delta\right)}{2\left(2-\beta+\beta\delta^2\right)\left(4-\beta+2\beta\delta^2\right)} > 0.$$

Putting $\bar{\kappa}_r = x_b$ concludes the proof. The entire picture can be seen in Fig. 2. □

Proof of Proposition 5. Here too condition (7) guarantees that the partially cooperative payoff is positive for some $k > 0$. Indeed, equation $U_{pc} = 0$ has two roots $k = 0$ and

$$k_b = \frac{(2-\beta)\left(-(2-\beta)(1-a) + \beta(1+a)\delta\right)}{4-\beta^2+\beta(2+3\beta)\delta^2},$$

which is positive if and only if (7) holds.

Consider first the deviation from the equilibrium Partial Cooperation (PC) to competition. Whoever has to remain idle (and empty) after the coin flip deviates by competitively purchasing; that is, simultaneously with the first-mover in the equilibrium. The harshest punishment consists in competing forever, and this punishment starts immediately. In this case, the deviating player faces competition payoff U_0 that starts from zero level and is conditional on the negative shock having already occurred:

$$U_0 = -B_2 + \beta V_{com}(k,k) = -B_2 + \frac{\beta}{4(1-\beta)}\big((2-\beta)A_2 - \beta B_2\big)$$

(where $V_{com}(k,k)$ is a payoff for competition when the profile is (k,k) – both players are full.) In Fig. 3, U_0 is drawn as the dashed red line. Instead, the equilibrium play delivers

$$U_- = \beta V_{pc}(0,k) = \frac{\beta}{16(1-\beta)}\left(2\beta(2-\beta)A_2 + \beta^2 A_1 - (2-\beta)(4-\beta)B_1\right),$$

where $V_{pc}(0,k)$ is a payoff for PC when the state profile is $(0,k)$ – the unit is empty unit and its opponent is full (see Proof of Prop. 1). In Fig. 3, U_- is drawn as the thin dashed blue line. This payoff differs from the PC line U_{pc}, because it is conditional on the already occurred negative shock and unfortunate outcome of the coin. The deviation is profitable if $U_0 > U_-$. Solving this inequality with respect to k, we obtain the condition

$$k < \underline{\kappa}_b = \frac{(8 - 8\beta + \beta^2)(-(2 - \beta)(1 - a) + \beta(1 + a)\delta)}{32 - 40\beta + 14\beta^2 - \beta^3 + \beta(16 - 16\beta + 3\beta^2)\delta^2},$$

with $\underline{\kappa}_b > 0$ as long as (7) holds. When $k < \underline{\kappa}_b$, PC is not an equilibrium: the player who has to wait her turn prefers to deviate to competition. When $k \geqslant \underline{\kappa}_b$ this particular deviation is not profitable. We still need to calculate all the Nash equilibria in that subgame off the equilibrium path, which we turn to later.

The payoff U_- plays an important role in finding the Partial Cooperation equilibrium. Even if the *ex ante* payoff U_{pc} is positive, a storage unit may find unprofitable to participate (under this equilibrium) after an adverse coin toss; that is, $U_- < 0$. Since U_- is a quadratic function of k with two roots $k = 0$ and some other nonzero root

$$k = \kappa_2 = \frac{(4 - \beta)(-(2 - \beta)(1 - a) + \beta(1 + a)\delta)}{8 - 6\beta + \beta^2 + \beta(8 - 3\beta)\delta^2},$$

inequality $U_- > 0$ is equivalent to $k < \kappa_2$. Note that $\kappa_2 < k_b$. For larger capacities, a deviation to competition may be either dominated or even no viable (i.e. negative payoff). Then another deviation consists in letting the opponent sell first. The deviating unit continues to play cooperatively when buying but reverts to ftl when selling. To compute a payoff from this deviation, we use the same tools as we did in Prop. 1. Namely, for cooperation+ftl, we have:

$$\mathbf{P} = \begin{pmatrix} -B_1/4 \\ -B_1/2 \\ A_1/2 \\ 0 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix},$$

$$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -2 & 0 & 3 \\ 1 & -2 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

Then, for $\beta < 1$ the payoff U_{colf} for cooperation+ftl is the following:

$$U_{colf} = V_{col+ftl}(0,0) = \frac{\beta\,(4 - 2\beta + \beta^2)\,A_1 - (8 - 4\beta^2 - \beta^3)\,B_1}{8(1-\beta)(2-\beta)(2+\beta)},$$

and this deviation is profitable when $U_{colf} > U_{pc}$. Note that we can calculate payoffs starting from state $(c_t^1 = 0, c_t^2 = 0)$ rather than from state $(c_{t'}^1 = k, c_{t'}^2 = k)$ because these strategies are the same between those two states. We get

$$U_{colf} > U_{pc} \quad \Leftrightarrow \quad k > \kappa_1 = \frac{-\beta(2+\beta)(1-a) + (8 - 4\beta - \beta^2)(1+a)\delta}{\beta(2+\beta) + (16 - 4\beta - 3\beta^2)\delta^2}.$$

The minimum of κ_1 and κ_2 gives us the value of $\overline{\kappa}_b$, above which ($k > \overline{\kappa}_b$) the PC equilibrium is not sustainable anymore. Hence, PC is an equilibrium for $k \in [\underline{\kappa}_b, \overline{\kappa}_b]$. Our findings are represented in Figure 3. There, κ_1 is the intersection of cooperation+ftl and PC payoff lines (dashed cyan line and solid blue line, respectively), and κ_2 is the point where the dashed blue line U_- turns from positive to negative. On the graph, $\kappa_1 < \kappa_2$, and so $\overline{\kappa}_b = \kappa_1$.

Since $\kappa_2 < k_b$, we conclude that $\overline{\kappa}_b < k_b$. Next we prove that $\underline{\kappa}_b < \overline{\kappa}_b$. We show $\underline{\kappa}_b < \overline{\kappa}_r$ first and then that $\overline{\kappa}_r < \overline{\kappa}_b$. When (7) holds,

$$\begin{aligned}
\overline{\kappa}_r - \underline{\kappa}_b &= \frac{\beta^2\,(2 + \beta\delta^2)\,\big(-(2-\beta)(1-a) + \beta(1+a)\delta\big)}{(4 - \beta + 2\beta\delta^2)\,(32 - 40\beta + 14\beta^2 - \beta^3 + \beta(16 - 16\beta + 3\beta^2)\delta^2)} > 0, \\
\kappa_1 - \overline{\kappa}_r &= \frac{(-2\beta(2+\beta) + (32 - 24\beta - 6\beta^2 + \beta^3)\,\delta^2)\,(1-a) + (2\,(16 - 12\beta - \beta^2) - \beta^2(4-\beta)\delta^2)\,(1+a)\delta}{(\beta(2+\beta) + (16 - 4\beta - 3\beta^2)\,\delta^2)\,(4 - \beta + 2\beta\delta^2)} \\
&> \frac{(-2\beta(2+\beta) + (32 - 24\beta - 6\beta^2 + \beta^3)\,\delta^2)\,(1-a) + (2\,(16 - 12\beta - \beta^2) - \beta^2(4-\beta)\delta^2)\,(1+a)\frac{2-\beta}{\beta}\frac{1-a}{1+a}}{(\beta(2+\beta) + (16 - 4\beta - 3\beta^2)\,\delta^2)\,(4 - \beta + 2\beta\delta^2)} \\
&= \frac{16(1-\beta)(1-a)}{\beta\,(\beta(2+\beta) + (16 - 4\beta - 3\beta^2)\,\delta^2)} > 0, \\
\kappa_2 - \overline{\kappa}_r &= \frac{(8 - 2\beta + \beta^2\delta^2)\,\big(-(2-\beta)(1-a) + \beta(1+a)\delta\big)}{(8 - 6\beta + \beta^2 + \beta(8 - 3\beta)\delta^2)\,(4 - \beta + 2\beta\delta^2)} > 0.
\end{aligned}$$

Hence, as long as (7) holds, $0 < \underline{\kappa}_b < \overline{\kappa}_b < k_b$; so the PC equilibrium *always* exists.

Of course, off the equilibrium path competition is not always an equilibrium for all $k \in (\underline{\kappa}_b, \overline{\kappa}_b)$ – as shown in Proposition 4. That is, upon observing a deviation (off the equilibrium path), players enter the punishment equilibrium; what is an equilibrium depends on the value of k. It is still Competition for $\underline{\kappa}_b < k \leqslant \overline{\kappa}_r$, but for $k \geqslant \overline{\kappa}_r$ the best response varies from ftl+competition to ftl to leaving the market immediately after selling energy. The next paragraphs conclude the proof with finding the conditions on k for SPNE out of equilibrium path.

Solving equation $U_{fc} = U_{ftl}$ with respect to k, we obtain the pivotal value

$$k_4 = \frac{-\beta^2(1-a) + (4-2\beta-\beta^2)(1+a)\delta}{\beta^2 + 2\left(4-\beta-\beta^2\right)\delta^2},$$

above which the ftl behavior becomes more profitable than ftl+competition. Note that this value is valid not only for state $c_t^1 = c_t^2 = 0$ but also for state $c_t^1 = c_t^2 = k$.

Next, solving for $U_{fc} = 0$ with respect to k, we obtain the pivotal value

$$k_o = \frac{-(2-\beta)(1-a) + \beta(1+a)\delta}{2-\beta+2\beta\delta^2},$$

above which trading zero quantities becomes more profitable than the ftl+competition behavior for state $c_t^1 = c_t^2 = 0$. In the case of state $c_t^1 = c_t^2 = k$, it means that selling k and quitting the market right afterwards is better than playing ftl+competition. For example, Figure 2 reflects the case $k_4 < k_o$.

It can be proven that $\min\{k_o, k_4\} \leqslant \overline{\kappa}_b$, whence we conclude that the behavior ftl+competition is always an equilibrium strategy for a deviating player if and only if $\overline{\kappa}_r \leqslant k \leqslant \min\{k_o, k_4\}$. The case $k_o < k_4$ does not allow any room for the ftl behavior because it becomes dominated by either ftl+competition or trading zero. As for $k_o \geqslant k_4$, ftl is the best response of a deviating unit for any k such that $k_4 < k < \min\{k_p, \overline{\kappa}_b\}$, where k_p is the nonzero root of equation $U_{ftl} = 0$:

$$k_p = \frac{-(4-2\beta-\beta^2)(1-a)+\beta^2(1+a)\delta}{4-2\beta-\beta^2+\beta^2\delta^2}.$$

k_p is a pivotal value starting from which never trading positive quantities (so that $c_t^1 = c_t^2 = 0$) or quitting (trading zero) right after selling all the energy ($c_t^1 = c_t^2 = k$) becomes more profitable than ftl. Then the deviating unit quits the market right after selling energy (either competitively or by "letting go") for any $k_o < k < \overline{\kappa}_b$ in the case $k_o < k_4$ and for any $k_p < k < \overline{\kappa}_b$ in the case $k_o \geqslant k_4$ and $k_p < \overline{\kappa}_b$. □

Proof of Proposition 6. We begin by ensuring that the cooperative payoff U_{col} is positive under condition (8). Equation $U_{col} = 0$ has two roots, $k = 0$ and

$$k_g = \frac{-(1-a)(2-\beta^2)+\beta(1+a)\delta}{2-\beta^2+\beta\delta^2}.$$

Inequality $U_{col} > 0$ is satisfied if and only if $k_g > 0$, which is equivalent to

$$\frac{1-a}{1+a} < \frac{\beta\delta}{2-\beta^2}. \tag{9}$$

It can be proven that condition (8) is stronger than (9) for any values of a, β, and δ.

Consider the deviations from Cooperation to competition, of which there can be multiple variations. That is, whoever has to remain idle after the coin flip, may deviate by either (competitively) buying or selling. The harshest punishment in this case is to compete forever, and it starts immediately. Consider first the case ($c_t^1 = 0, c_t^2 = 0$) – both units are empty. Then the deviating player faces competition payoff U_0 that starts from $(0, 0)$, and is conditional on the negative shock having already occurred:

$$U_0 = -B_2 + \beta V_{com}(k,k) = -B_2 + \frac{\beta}{4(1-\beta)}\big((2-\beta)A_2 - \beta B_2\big),$$

where $V_{com}(k,k)$ is the continuation payoff for competition in state (k,k) – both players are full. The alternative is not to deviate from the equilibrium prescription and wait to buy, which delivers

$$U_{-}^{c}=\beta V_{col}(0,k)=\frac{\beta}{4\left(2-\beta-\beta^{2}\right)}\left(\beta(1+\beta)A_{1}-\left(4-\beta-\beta^{2}\right)B_{1}\right),$$

where $V_{col}(0,k)$ is the continuation payoff from cooperation for an empty unit when its opponent is full – state $(0,k)$ (see Proof of Prop. 1). The deviation is unprofitable if $U_0 < U_{-}^{c}$. Solving this inequality with respect to k, we obtain

$$k<\kappa_{0}=\frac{-\left(8-8\beta-\beta^{2}+2\beta^{3}\right)(1-a)+\beta\left(4-\beta-2\beta^{2}\right)(1+a)\delta}{16-12\beta-3\beta^{2}+3\beta^{3}+\beta(8-\beta-2\beta^{2})\delta^{2}}.$$

Suppose the profile of charge is $(c_t^1 = k, c_t^2 = k)$. Deviating yields the competition payoff U_k conditional on the positive shock:

$$U_{k}=A_{2}+\beta V_{com}(0,0)=A_{2}+\frac{\beta}{4(1-\beta)}\left(\beta A_{2}-(2-\beta)B_{2}\right),$$

where $V_{com}(0,0)$ is a payoff for competition when both players are empty (state $(c_t^1 = 0, c_t^2 = 0)$). If playing equilibrium instead, one receives

$$U_{+}^{c}=\beta V_{col}(k,0)=\frac{\beta}{4\left(2-\beta-\beta^{2}\right)}\left(\left(4-\beta-\beta^{2}\right)A_{1}-\beta(1+\beta)B_{1}\right),$$

where $V_{col}(k,0)$ is a payoff for cooperation when the state is $(c_t^1 = k, c_t^2 = 0)$ (see Proof of Prop. 1). The deviation is unprofitable if $U_k < U_{+}^{c}$. Solving this inequality with respect to k, we obtain

$$k<\kappa_{k}=\frac{-\beta\left(4-\beta-2\beta^{2}\right)(1-a)+\left(8-8\beta-\beta^{2}+2\beta^{3}\right)(1+a)\delta}{\beta(8-\beta-2\beta^{2})+\left(16-12\beta-3\beta^{2}+3\beta^{3}\right)\delta^{2}}.$$

Next, compute

$$\kappa_{k}-\kappa_{0}=\frac{2(1-\beta)(2+\beta)K(a,\beta,\delta)}{\left(\beta(8-\beta-2\beta^{2})+\left(16-12\beta-3\beta^{2}+3\beta^{3}\right)\delta^{2}\right)\left(16-12\beta-3\beta^{2}+3\beta^{3}+\beta(8-\beta-2\beta^{2})\delta^{2}\right)},$$

where

$$\begin{aligned} K(a,\beta,\delta) &= \left(-(2-\beta)\beta^2 + \left(32 - 40\beta + 2\beta^2 + 7\beta^3\right)\delta^2\right)(1-a) \\ &+ \left(32 - 40\beta + 2\beta^2 + 7\beta^3 - (2-\beta)\beta^2\delta^2\right)(1+a)\delta. \end{aligned}$$

Using (9), we obtain

$$\begin{aligned} K(a,\beta,\delta) &> \left(-(2-\beta)\beta^2 + \left(32 - 40\beta + 2\beta^2 + 7\beta^3\right)\delta^2\right)(1-a) \\ &+ \left(32 - 40\beta + 2\beta^2 + 7\beta^3 - (2-\beta)\beta^2\delta^2\right)(1+a)\frac{2-\beta^2}{\beta}\frac{1-a}{1+a} \\ &= \frac{(1-\beta)(1-a)}{\beta}\left(64 - 16\beta - 44\beta^2 + 8\beta^3 + 7\beta^4 + \beta\left(32 - 12\beta - 8\beta^2 + \beta^3\right)\delta^2\right) \\ &> 0. \end{aligned}$$

Thus $\kappa_0 < \kappa_k$ if (9) holds, and any deviation to competition is unprofitable only if $k > \kappa_k$. Putting $\underline{\kappa}_g = \kappa_k$ finishes the first part of the proof.

In Fig. 4, U_k is drawn as the thin dashed red line, and U_+^c is the highest of the two thin dashed green arches. The intersection of these lines is exactly $\underline{\kappa}_g$.

Even if (9) holds, it may become unprofitable to stick to cooperation if moving second, that is, when $U_-^c < 0$. Since U_-^c is a quadratic function of k with two roots $k = 0$ and some other nonzero root

$$\overline{\kappa}_g = \frac{-(4-\beta-\beta^2)(1-a) + \beta(1+\beta)(1+a)\delta}{4-\beta-\beta^2+\beta(1+\beta)\delta^2},$$

inequality $U_-^c > 0$ is equivalent to $k < \overline{\kappa}_g$. Note that $\overline{\kappa}_g < k_g$.

In Fig. 4, U_-^c is drawn as the lowest of the two thin dashed green arches, and intersects the horizontal axe at $\overline{\kappa}_g$. In the small interval $(\overline{\kappa}_g, k_g)$, cooperation still delivers a positive payoff *on average*, but the second mover stops immediately; that is, cooperation is not an equilibrium. Hence, the Cooperation equilibrium exists in the interval $(\underline{\kappa}_g, \overline{\kappa}_g)$, if this interval exists. Indeed,

$$\overline{\kappa}_g - \underline{\kappa}_g = \frac{(2+\beta)\left(-\left(G_3(\beta)+G_4(\beta)\delta^2\right)(1-a)+\left(G_1(\beta)+G_2(\beta)\delta^2\right)(1+a)\delta\right)}{\left(4-\beta-\beta^2+\beta(1+\beta)\delta^2\right)\left(\beta(8-\beta-3\beta^2)+(16-12\beta-3\beta^2+3\beta^3)\delta^2\right)},$$

and $\underline{\kappa}_g < \overline{\kappa}_g$ is equivalent to condition (8).

Of course, competition is not always an equilibrium for all $k \in (\underline{\kappa}_g, \overline{\kappa}_g)$ along the off-equilibrium path after deviation, and the best response to the punishment in this subgame may be different, similar to what we observe in the proof of Proposition 5. The best response of the deviating unit depends on the order of parameters $\underline{\kappa}_g$, $\overline{\kappa}_g$, $\overline{\kappa}_r$, k_4, k_o, and k_p and may result in playing competition, or ftl+competition, or ftl, or even quitting the market after selling all the energy. □

Proof of Corollary 7. The existence of the equilibrium Cooperation is guaranteed by condition (8), so we seek conditions for which it is known to hold. The LHS of (8) is always positive, so we need to guarantee the RHS is also positive; in that case, there is a value of a such that (8) holds. Since the denominator of the RHS is always positive, we just need to find the conditions that provides $G_1(\beta) + G_2(\beta)\delta^2 > 0$. Note that $G_2 > 0$ for any β, so we can put $\delta = 1$. Then we have

$$G_1(\beta) + G_2(\beta) = 4(-4 + 8\beta - 2\beta^2 - \beta^3),$$

which is positive if $\beta > \beta^* = 0.6309$. □

Proof of Proposition 8. The second part is obvious. If (8) fails to hold, cooperation can never be an equilibrium behavior for any k. Even if (8) holds but $k_3 > \overline{\kappa}_g$, $U_{pc} > U_{col}$ for any $k \in [\underline{\kappa}_g, \overline{\kappa}_g]$, so while playing cooperation is an equilibrium, it cannot be payoff-maximizing. Competition is the only option for $k \in (0, \underline{\kappa}_b]$, and starting from $\underline{\kappa}_b$, $U_{pc} > U_{com}$ until $\overline{\kappa}_b$, when partial cooperation stops being an equilibrium.

To prove the first case, it is enough to show that $\underline{\kappa}_g < k_3$ under (8) and high enough β. We have

$$k_3 - \underline{\kappa}_g = \frac{(2+\beta)(1+a)\bar{K}(\frac{1-a}{1+a}, \beta, \delta)}{\left(\beta^2+(12-2\beta-3\beta^2)\delta^2\right)\left(\beta(8-\beta-3\beta^2)+(16-12\beta-3\beta^2+3\beta^3)\delta^2\right)},$$

where

$$\bar{K}\left(\frac{1-a}{1+a}, \beta, \delta\right) = -\beta\Big((2-\beta)\beta^2 - (24-30\beta+4\beta^2+3\beta^3)\delta^2\Big)\frac{1-a}{1+a} + \Big(\beta(16-22\beta+6\beta^2+\beta^3) - (2-\beta)(8-8\beta-2\beta^2+3\beta^3)\delta^2\Big)\delta.$$

According to (8), we have

$$0 < \frac{1-a}{1+a} < \frac{G_1(\beta)+G_2(\beta)\delta^2}{G_3(\beta)+G_4(\beta)\delta^2} \cdot \delta.$$

Since the function $\bar{K}$ is linear in $(1-a)/(1+a)$, it is enough to show that $\bar{K}$ is positive at the ends of the interval. We have

$$\begin{aligned}\bar{K}(0,\beta,\delta) &= \big(\beta(16-22\beta+6\beta^2+\beta^3) - (2-\beta)(8-8\beta-2\beta^2+3\beta^3)\delta^2\big)\,\delta \\ &\geqslant 2(1-\beta)(-8+12\beta-\beta^2-2\beta^3),\end{aligned}$$

which is positive if $\beta \geqslant 0.81$. Also,

$$\bar{K}\left(\frac{G_1(\beta)+G_2(\beta)\delta^2}{G_3(\beta)+G_4(\beta)\delta^2}\cdot\delta, \beta, \delta\right) = \frac{2(1-\beta)(2-\beta)\delta\Big(\beta(4-\beta)-(8-6\beta-3\beta^2)\delta^2\Big)\Big(\beta(8-\beta-3\beta^2)+(16-12\beta-3\beta^2+3\beta^3)\delta^2\Big)}{(2-\beta)\beta(4-\beta-\beta^2)+(32-48\beta+14\beta^2+5\beta^3-\beta^4)\delta^2}.$$

Since

$$\beta(4-\beta)-(8-6\beta-3\beta^2)\delta^2 \geqslant 2(-4+5\beta+\beta^2)$$

and the last polynomial is positive if $\beta \geqslant 0.71$, we may conclude that $k_3 - \underline{\kappa}_g \geqslant 0$ at least for all $\beta \geqslant 0.81$. This is laid out in Figure 5 (for $k_3 \leqslant \overline{\kappa}_g$) and Figure 6 (for $k_3 > \overline{\kappa}_g$). The solid lines depict equilibrium maximum payoffs that arise from the equilibria listed in Proposition 8. The dashed lines show payoffs arising from the same equilibria, but are payoff-dominated by another equilibrium. For example, in Figure 5, between $\underline{\kappa}_g$ and k_3, Cooperation is an equilibrium but it is dominated by Partial Cooperation, which delivers higher payoffs.

This is reversed for k larger than k_3. In both Figures, at $\underline{\kappa}_b$, Partial Cooperation must deliver a discretely larger payoff than competition otherwise the deviation is too tempting (equivalently, the benefit of Cooperation too small). This reflects the fact that Partial Cooperation must be robust to deviations at the interim stage – when one of the players is revealed to be the second mover and contemplates her options then (see Section 3.3). However, in Figure 5, storage operators are indiffirent between either cooperative equilibrium at k_3. At that point, the ex ante incentives are relevant. □

Proof of Proposition 11. We use the same algebra as in Prop. 1. For competition, the system of equations (6) takes the following form:

$$\begin{cases} V(0,0) = \frac{1}{2}\left(\beta V(0,0) - B_2\left(\frac{k}{2}\right) + \beta V\left(\frac{k}{2},\frac{k}{2}\right)\right), \\ V\left(\frac{k}{2},\frac{k}{2}\right) = \frac{1}{2}\left(A_2\left(\frac{k}{2}\right) + \beta V(0,0) - B_2\left(\frac{k}{2}\right) + \beta V\left(k,k\right)\right), \\ V(k,k) = \frac{1}{2}\left(A_2\left(\frac{k}{2}\right) + \beta V\left(\frac{k}{2},\frac{k}{2}\right) + \beta V(k,k)\right), \end{cases}$$

so (now omitting the argument $k/2$ in all A_i and B_i throughout the proof)

$$\mathbf{P} = \frac{1}{2}\begin{pmatrix} -B_2 \\ A_2 - B_2 \\ A_2 \end{pmatrix}, \qquad \mathbf{Q} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

To calculate the power t of matrix $\mathbf{Q}$, we find the Jordan decomposition $\mathbf{Q} = \mathbf{T}\cdot\mathbf{J}\cdot\mathbf{T}^{-1}$ of $\mathbf{Q}$. Here,

$$\mathbf{J} = \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & -1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

so $\mathbf{Q}^t = \mathbf{T}\cdot\mathbf{J}^t\cdot\mathbf{T}^{-1}$.

For $\beta < 1$, we obtain:

$$\begin{aligned}\mathbf{V} &= \mathbf{P} + \sum_{i=1}^{\infty} \beta^i \mathbf{Q}^i \cdot \mathbf{P} = \mathbf{P} + \mathbf{T} \cdot \begin{pmatrix} -\frac{\beta}{2+\beta} & 0 & 0 \\ 0 & \frac{\beta}{2-\beta} & 0 \\ 0 & 0 & \frac{\beta}{1-\beta} \end{pmatrix} \cdot \mathbf{T}^{-1} \cdot \mathbf{P} \\ &= \frac{1}{(1-\beta)(2-\beta)(2+\beta)} \begin{pmatrix} \beta A_2 - (2-\beta^2) B_2 \\ (2-\beta)(A_2 - B_2) \\ (2-\beta^2) A_2 - \beta B_2 \end{pmatrix}.\end{aligned}$$

The uppermost term is exactly $V(0,0) = U_{com}$.

For cooperation and partial cooperation, there are not three states anymore but nine. The elements of (6) are

$$\mathbf{V} = \begin{pmatrix} V(0,0) \\ V(0,k/2) \\ V(0,k) \\ V(k/2,0) \\ V(k/2,k/2) \\ V(k/2,k) \\ V(k,0) \\ V(k,k/2) \\ V(k,k) \end{pmatrix}, \quad \mathbf{P}_{col} = \frac{1}{4} \begin{pmatrix} -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ A_1 - B_1 \\ A_1 - 2B_1 \\ 2A_1 \\ A_1 \\ A_1 \end{pmatrix}, \quad \mathbf{Q}_{col} = \frac{1}{4} \begin{pmatrix} 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \end{pmatrix},$$

$$\mathbf{P}_{pc} = \frac{1}{4}\begin{pmatrix} -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - 2B_1 \\ 2A_1 \\ 2A_2 \\ 2A_2 \end{pmatrix}, \qquad \mathbf{Q}_{pc} = \frac{1}{4}\begin{pmatrix} 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \end{pmatrix}.$$

Again we find the payoffs using Jordan's decomposition. For example, for cooperation the corresponding Jordan form and transition matrix are

$$\mathbf{J}_{col} = \frac{1}{4}\begin{pmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1-\sqrt{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-\sqrt{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}-1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1+\sqrt{5} \end{pmatrix},$$

$$\mathbf{T}_{col} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & 0 & -1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{1+\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 2 & 0 & 1 & -2 & 1 & 1+\sqrt{5} & 0 & 1-\sqrt{5} & 0 \\ 1 & -1 & 0 & 1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{1+\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 0 & 0 & -1 & 0 & 1 & 1+\sqrt{5} & 0 & 1-\sqrt{5} & 0 \\ -1 & -1 & 0 & -1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{-1-\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{-1+\sqrt{5}}{2} \\ -2 & 0 & 1 & 2 & 1 & 1+\sqrt{5} & 0 & 1-\sqrt{5} & 0 \\ 1 & 1 & 0 & 1 & 1 & \frac{-3-\sqrt{5}}{2} & \frac{-1-\sqrt{5}}{2} & \frac{-3+\sqrt{5}}{2} & \frac{-1+\sqrt{5}}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Using the same steps as we did for competition, we can obtain U_{col}. We repeat the algorithm for the payoff function U_{pc} but omit this last iteration here to conserve space. □

Proof of Proposition 12. Here too we use the same algorithm as for proving Proposition 11. Now we have four states in competition (that we list below), and sixteen states for cooperation and partial cooperation. For competition, the system of equations (6) takes the following form:

$$\begin{cases} V(0,0) = \frac{1}{2}\left(\beta V(0,0) - B_2\left(\frac{k}{3}\right) + \beta V\left(\frac{k}{3}, \frac{k}{3}\right)\right), \\ V\left(\frac{k}{3}, \frac{k}{3}\right) = \frac{1}{2}\left(A_2\left(\frac{k}{3}\right) + \beta V(0,0) - B_2\left(\frac{k}{3}\right) + \beta V\left(\frac{2k}{3}, \frac{2k}{3}\right)\right), \\ V\left(\frac{2k}{3}, \frac{2k}{3}\right) = \frac{1}{2}\left(A_2\left(\frac{k}{3}\right) + \beta V(\frac{k}{3}, \frac{k}{3}) - B_2\left(\frac{k}{3}\right) + \beta V\left(k,k\right)\right), \\ V(k,k) = \frac{1}{2}\left(A_2\left(\frac{k}{3}\right) + \beta V\left(\frac{2k}{3}, \frac{2k}{3}\right) + \beta V(k,k)\right), \end{cases}$$

so (now omitting the argument $k/3$ in all A_i and B_i throughout the proof)

$$\mathbf{P} = \frac{1}{2}\begin{pmatrix} -B_2 \\ A_2 - B_2 \\ A_2 - B_2 \\ A_2 \end{pmatrix}, \qquad \mathbf{Q} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

Here the Jordan decomposition of $\mathbf{Q}$ is $\mathbf{Q} = \mathbf{T} \cdot \mathbf{J} \cdot \mathbf{T}^{-1}$:

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1/\sqrt{2} \end{pmatrix}, \qquad \mathbf{T} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1+\sqrt{2} & 1-\sqrt{2} \\ -1 & 1 & -1-\sqrt{2} & -1+\sqrt{2} \\ 1 & 1 & 1 & 1 \end{pmatrix},$$

and its power t is given by $\mathbf{Q}^t = \mathbf{T} \cdot \mathbf{J}^t \cdot \mathbf{T}^{-1}$. For $\beta < 1$, we obtain:

$$\mathbf{V} = \mathbf{P} + \sum_{i=1}^{\infty} \beta^i \mathbf{Q}^i \cdot \mathbf{P} = \mathbf{P} + \mathbf{T} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{\beta}{1-\beta} & 0 & 0 \\ 0 & 0 & -\frac{\beta}{\sqrt{2}+\beta} & 0 \\ 0 & 0 & 0 & \frac{\beta}{\sqrt{2}-\beta} \end{pmatrix} \cdot \mathbf{T}^{-1} \cdot \mathbf{P} =$$

$$= \frac{1}{8(1-\beta)(2-\beta^2)} \begin{pmatrix} \beta(4-\beta^2)A_2 - (8-4\beta^2-\beta^3)B_2 \\ (2+\beta)(2-\beta)^2 A_2 - (8-6\beta^2+\beta^3)B_2 \\ (8-6\beta^2+\beta^3)A_2 - (2+\beta)(2-\beta)^2 B_2 \\ (8-4\beta^2-\beta^3)A_2 - \beta(4-\beta^2)B_2 \end{pmatrix}.$$

The uppermost term is exactly $V(0,0) = U_{com}$.

For cooperation and partial cooperation, there are 16 states possible. The elements of (6) are

$$\mathbf{V} = \begin{pmatrix} V(0,0) \\ V(0,k/3) \\ V(0,2k/3) \\ V(0,k) \\ V(k/3,0) \\ V(k/3,k/3) \\ V(k/3,2k/3) \\ V(k/3,k) \\ V(2k/3,0) \\ V(2k/3,k/3) \\ V(2k/3,2k/3) \\ V(2k/3,k) \\ V(k,0) \\ V(k,k/3) \\ V(k,2k/3) \\ V(k,k) \end{pmatrix}, \qquad \mathbf{P}_{col} = \frac{1}{4} \begin{pmatrix} -B_1 \\ -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ A_1 - B_1 \\ A_1 - B_1 \\ A_1 - 2B_1 \\ 2A_1 - B_1 \\ A_1 - B_1 \\ A_1 - B_1 \\ A_1 - 2B_1 \\ 2A_1 \\ A_1 \\ A_1 \\ A_1 \end{pmatrix}, \qquad \mathbf{P}_{pc} = \frac{1}{4} \begin{pmatrix} -B_1 \\ -B_1 \\ -B_1 \\ -2B_1 \\ 2A_1 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - 2B_1 \\ 2A_1 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - B_1 \\ 2A_2 - 2B_1 \\ 2A_1 \\ 2A_2 \\ 2A_2 \\ 2A_2 \end{pmatrix},$$

$$
\mathbf{Q}_{col} = \frac{1}{4}\begin{pmatrix}
2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 2
\end{pmatrix},
$$

$$
\mathbf{Q}_{pc} = \frac{1}{4}\begin{pmatrix}
2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2
\end{pmatrix}.
$$

Here too we omit the repetition of the algorithm for the functions U_{col} and U_{pc}. □

References

Abreu, Dilip (1988) "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, 56 (3), 383–396.

Acemoglu, Daron, Ali Kahkbod, and Asuman Ozdaglar (2017) "Competition in Electricity Markets with Renewable Energy Sources.," *The Energy Journal*, 38, 137–155.

Andres-Cerezo, A. and Natalia Fabra (2023a) "Storage and Renewables Energies: Friends of Foes," *Working Paper*.

——— (2023b) "Storing power: market structure matters," *Rand Journal of Economics*, 54 (1), 3–53.

Balakin, Sergei and Guillaume Roger (2023) "Near-opitmal storage strategies in electricity market," *working paper*.

Bonatti, A., G. Cisternas, and Juuso Toikka (2017) "Dynamic Oligopoly with Incomplete Information," *The Review of Economic Studies*, 84, 503–546.

Borenstein, Severin and James Bushnell (1999) "An empirical analysis of the potential for market power in California's electricity industry.," *Journal of Industrial Economics*, 47 (3), 285–323.

Borenstein, Severin, James Bushnell, and C. R. Knittel (1999) "Market Power in Electricity Markets: Beyond Concentration Measures.," *Energy Journal*, 20 (4).

Bushnell, James, Erin Mansur, and Celeste Saravia (2008) "Vertical Arrangements, Market Structure and Competition: An analysis of Restructured U.S. Electricity Markets.," *The American Economic Review*, 98 (1).

Butters, R. Andrew, Jackson Dorsey, and Gautam Gowrisankaran (Working Paper) "Soaking Up the Sun: Battery Investment, Renewable Energy, and Market Equilibrium.," *R&R Econometrica*.

Deaton, Angus and Guy Laroque (1992) "On the Behaviour of Commodity Prices," *The Review of Economic Studies*, 59 (1), 1–23.

Dávila, Eduardo, Daniel Graves, and Cecilia Parlatore (2024) "The Value of Arbitrage," *Journal of Political Economy*, forthcoming.

Geske, J. and Richard Green (2020) "Optimal Storage, Investment and Management under Uncertainty: It is Costly to Avoid Outages!," *Energy Journal*, 41 (SI).

Gromb, Denis and Dimitri Vayanos (2002) "“Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs," *The Journal of Financial Economics*, 66 (2), 361–407.

Harrison, J. M. and Alison Taylor (1978) "Optimal control of a Brownian storage system," *Stochastic processes and their applications*, 6, 179–194.

Karaduman, Omer (2020) "The economics of grid-scale storage," *mimeo*, Stanford University.

Klemperer, P. and Margaret Meyer (1989) "Supply function equilibria in oligopoly under uncertainty," *Econometrica*, 57 (6), 1243–1277.

Lundin, Erik and Thomas P. Tangerås (2020) "Cournot competition in wholesale electricity markets: The Nordic power exchange, Nord Pool.," *International Journal of Industrial Organization*, 68.

Oehmke, Martin (2009) "Gradual Arbitrage," *working paper.*

Samuelson, Paul (1971) "Stochastic Speculative Price," *Proceedings of the National Academy of Sciences*, 68 (2), 335–337.

Sannikov, Yulyi and Andrzej Skrzypacz (2007) "Impossibility of Collusion under Imperfect Monitoring with Flexible Production," *American Economic Review*, 97 (5), 1794–1823.

Shleifer, Andrei and Robert Vishny (1997) "The Limits of Arbitrage," *The Journal of Finance*, 52 (1), 35–55.

Vayanos, Dimitri (1999) "Strategic trading and welfare in a dynamic market," *The Review of Economic Studies*, 66, 219–254.

Willems, Bert, Ina Rumiantseva, and Hannes Weigt (2009) "Cournot versus Supply Functions: What does the data tell us?" *Energy Economics*, 31 (1), 38–47.

Williams, Olayinka and Richard Green (2022) "Electricity storage and market power," *Energy Policy*, 164.

Wright, Brian D and Jeffrey C. Williams (1984) "The Welfare Effects of Storage," *The Quarterly Journal of Economics*, 99 (1), 169–192.