Preemption equilibrium in a storage game

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Abstract

We study a game with a finite number of incumbent sellers and a new agent that is storage; storage must buy to sell. The immediate, but not exclusive, application is electricity markets. We construct an equilibrium in which the incumbents cooperate to prevent storage from operating, even absent entry costs. The producers achieve cooperation through the threat of Nash reversion and preempt operation of the storage unit with the threat to cycle between high purchase prices and low selling prices. Because two kinds of deviations must be deterred, the equilibrium requires a two-sided condition on the discount factor. Market power is essential to the result, which fits a strategic environment, as is the need of storage to first buy. Combined, these characteristics allow other players to determine the marginal cost of storage.

Key words: *stochastic game, dynamic trading, energy storage*

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1 Introduction

In electricity markets, the emergence of storage is transformative. It may render the energy transition feasible by making energy available when need rather than when produced and so acts as a complement to variable renewable energy sources. However storage also disrupts trading as it uses inherently dynamic strategies; it must first buy to then sell, and can do so at any time it finds opportune. And for conventional generators that realize most of their payoffs during high-price periods, storage is an unwelcome competitor.

In this paper we study a pointed question. We demonstrate that conventional producers (in electricity, thermal generators) can (collectively) prevent a storage unit from operating. That is, storage entry into a market dominated by conventional production-for-sale is not a foregone conclusion. In terms of policy, this suggests that competition authorities should monitor the behavior of large producers to ensure entry is not preempted. The key point of our result is that the marginal cost of storage is determined by the behavior of *all* players when buying. This differs from any game studied so far, especially more standard problems of selling only, where the marginal cost is exogenous and only revenue is endogenously determined.[1] The reason is that storage must buy in order to sell–it feeds on an arbitrage spread–and players have market power. Therefore, this first purchase can act like an entry cost that may not be covered by the selling revenue. A policy maker can intervene by offering a subsidy that equals the revenue shortfall. However, and this is a direct consequence of the equilibrium construction, this subsidy should be paid out each period into perpetuity.

Technically, we study a long-horizon game with a finite number of conventional sellers (equivalently, producers) and one storage operator, all playing a quantity game. They face a stochastic demand; the aggregate demand shocks induce a sequence of high and low prices over time. A storage operator can step in to exploit these price differences by implementing the simple idea of "buying low and selling high"; this is known as *merchant storage*. This sequence of events defines a stochastic game, which admits many equilibria (Shapley (1953)). We focus on constructing a subgame-perfect equilibrium that is novel, and in which the storage

[1] In more general games, the cost of an action is exogenous–including if zero.

operator never finds it profitable to trade. The reason is that low (buying) and high (selling) prices are determined endogenously by the actions of all players. The conventional sellers can collude to keep "low" prices sufficiently high to be unattractive for the storage unit to buy, given that it must subsequently sell under conditions they also determine as part of the equilibrium play. Any deviation, by any player, triggers Nash reversion for at least the next selling opportunity; it is this off-equilibrium play that deters buying in the first place. The novelty of this equilibrium is the combination of a simple grim-trigger strategy that supports cooperation between producers with a cycling play that deters the first purchase by the storage unit. Disciplining the producers is achieved using Nash reversion as the grim-trigger strategy. In the cycle, producers can observe whether storage buys (at a monopoly price) to adjust their actions when selling (at a Cournot price). In equilibrium, buying is never worthwhile for the storage unit. We call this a "preemption equilibrium", and explain it in detail in Section 3.2.[2] Like any cooperative equilibrium, the discount factor must be large enough for grim-trigger to have bite. But, in a novel twist, this discount factor may also be bounded from above (below 1) to ensure the one-cycle payoff to storage remains negative: the off-path expected Cournot revenue must remain low enough to make the trade unattractive to storage. For this equilibrium to exist, two elements are required: the storage unit must first buy (to then sell) and the purchase price must be determined by equilibrium play (i.e. the other players). That is, market power is a key ingredient; this must be a strategic game. We also speak to the robustness of this equilibrium.

This paper sits at the intersection of game theory and the economics of electricity. However its relevance extends beyond electricity markets: it applies to any environment where merchant storage can be active, including market making in securities.[3] It contributes to the extant literature on electricity markets because it seeks to characterize behavior in a stochastic environment with market power. Karaduman (2020) studies grid scale storage. Generators and the storage unit play an infinite horizon game and market power is internalized. However, Karaduman (2020) limits himself to simulating the best reply from the data. So, the actual

[2] It may not be the only equilibrium that results in no trade for the storage unit.

[3] A market maker buys, holds and sell securities strategically, and has a capacity bounded by its own capital.

behavior of the storage unit is never known. Andres-Cerezo and Fabra (2023) study market structure with storage, but do not delve into how storage actually behaves. A generator can enhance its market power by also owning storage, especially when demand is the highest: the joint ownership of these two assets induces more quantity withholding. In their model, this is not tacit collusion but simply joint-profit maximization of two assets owned by the same controlling entity.

Shapley (1953) introduced stochastic games, which have been used in economics to model entry (and exit) decisions in industrial organization (Pakes and McGuire (1994), Doraszelski and Satterthwaite (2010) and others). Their study has led Maskin and Tirole (2001) to develop the solution concept of a Markov perfect equilibrium- (MPE)–see also Doraszelski and Escobar (2010) in particular, and many others, since. In their paper on repeated oligopoly under incomplete information, Bonatti et al. (2017) rely on the MPE concept; their focus is on learning. In contrast we construct a subgame-perfect equilibrium of our game since we want to study a cooperative equilibrium, the enforcement of which depends on the history of the game. That is, players must have at least some information about the entire history of play for the equilibrium to be sustainable through off-path punishment threats. In a paper concerned with providing foundations for Markov perfection, Bhaskar et al. (2013) also point out that the cost of the simplicity and descriptive accuracy of MPEs is the loss of payoff-irrelevant histories that are strategically useful; some equilibria are lost. Here we exploit these off-path histories to support our equilibrium. The conditions of Bhaskar et al. (2013) do not apply to our equilibrium, which requires perfect recall.

2 Model

Consider a market with one storage unit, n producers (sellers) indexed by $j = 1, 2, ...n$, and a pool of consumers. The set of (strategic) players is labeled $\mathcal{N}$. Consumers are diffuse and not strategic. Their behavior is described by the demand function $D(p_t, \varepsilon_t)$ for each period t, where $\varepsilon_t \in \mathcal{E}$ is a shock distributed according to some commonly known distribution F. Producers are strategic players. Each of them produces a quantity q_t^j (for example, energy) for each period t, so $\mathbf{q}_t$ is the action vector; they are not subject to capacity constraints.

The storage unit is a special player in this game with a finite capacity k. In each period t, the storage operator can either buy b_t (for example, energy) up to its capacity, or sell s_t. We specialize the model in that $s, b \in \{0, k\}$; the storage buys or sell its entire capacity each time.[4] This process can be described by a simple equation of motion:

$$c_t = c_{t-1} + b_t - \frac{s_t}{\delta}, \qquad t \in \mathbb{N}, \qquad c_0 = 0;\ c_t \in \{0, k\},\ t > 0. \tag{1}$$

Here, c_t is the current inventory level, δ is a "round-trip" efficiency parameter ($0 < \delta \leqslant 1$), and $b_t \geqslant 0, s_t \geqslant 0$. We suppose the storage unit has a discount factor $\beta < 1$; it is exposed to a strictly positive interest rate. A storage operator can only either buy or sell in each period, so $b_t \cdot s_t = 0$ for any t. This is an assumption (in electricity, a technical characteristic), but we also note that it cannot be optimal to simultaneously buy and sell; with $\delta < 1$, it is even strictly suboptimal. The set of actions is denoted $\mathcal{A}_t := \left\{\mathbf{q}_t \in \mathbb{R}^n_+;\ b_t, s_t \in \{0, k\}\right\}$. Sellers bid quantities in a centralized market; this market clears if

$$D(p_t, \varepsilon_t) = \sum_{j=1}^{n} q_t^j - b_t + s_t \tag{2}$$

for any t. Since the nature of competition is not the primary object of interest, throughout the rest of the paper we consider a linear demand function:

$$D(p_t, \varepsilon_t) = 1 - p_t + \varepsilon_t.$$

The exact timing of the game is as follows: the storage unit starts empty, and at each moment t, a (invisible) market operator collects consumer demand $D(p_t, \varepsilon_t)$. All players observe the shock ε_t and storage moves first to either buy or sell. Producers bid q_t^j for all $j = 1, 2, ..., n$. Of course, storage (correctly) anticipates their response (in equilibrium). For completeness, the storage operator can bid more than the suppliers are willing to offer–for example, $\mathbf{q} = \mathbf{0}$.

[4] This is a special version of the model of our companion paper, in which $s, b \in [0, k]$; see Balakin and Roger (2025a).

Producers each maximize their payoffs

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t p_t q_t^j\right], \tag{3}$$

by choice ofq_t^j for each $t \geq 0$ and $j \in \mathcal{N}$. The objective of the storage operator is to maximize

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t p_t (s_t - b_t)\right], \tag{4}$$

over $s_t, b_t,\ t \geq 0$ and subject to the law of motion (1).

For our purposes in studying this game, the history of play matters. Therefore we need to work with the concept of subgame-perfection. The state variables are a pair of inventory levels and demand shocks, so actions are mappings $b_t, s_t : \mathcal{C} \times \mathcal{E} \times \mathcal{H}_t \mapsto \{0, k\}$, where $\mathcal{C} := \{0, k\}$ and $\mathcal{H}_t$ is the set of all histories at time t, with $\mathcal{H}_0 \equiv \emptyset$. Because actions $b_t := b(c_t, \varepsilon_t; H_t)$ and $s_t := s(c_t, \varepsilon_t; H_t)$ already encode the state (c, ε) of the system, histories $H_t \in \mathcal{H}_t$ are constructed in standard fashion. For the storage unit, a strategy is a sequence of these actions from time 0 to ∞. The corresponding value function reads

$$V(c) = \sup_{b,s}\ \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t p_t (s_t - b_t)\right].[5] \tag{5}$$

We denote the set of payoffs by $\mathcal{U}$; this set is to be understood as all the possible values that (3) and (4) can take.

Definition 1. *An equilibrium of the game* $\mathcal{G} := \{F, \mathcal{N}, \mathcal{A}, \mathcal{U}\}$ *is*

- *a sequence of vectors* $\{\mathbf{q}^*{}_t\}_{t=0}^{\infty}$ *such that, for each* j,

$$\{(q^*)_t^j\}_{t=0}^{\infty} \in \arg\max \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t p_t q_t^j\right];$$

[5]We dispense proving that the Dynamic Programming Principle holds in this environment, which is quite standard.

- *a sequence of pairs* $\{b_t^*, s_t^*\}_{t=0}^{\infty}$ *such that*

$$\{b_t^*, s_t^*\}_{t=0}^{\infty} \in \arg\max \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t p_t(s_t - b_t)\right]$$

- *and for all* $t = 0, 1, .., \infty$ *the market clears:* (2) *holds.*

We confine our attention to symmetric equilibria. Depending on the decisions of the storage operator, in each round there may be either

- n (symmetric) active players; or

- $n + 1$ active players, including the storage unit.

Next we explain trading in the dynamic game and then turn to the object of this paper, which is the construction of the exclusion equilibrium.

3 The result

For later reference we present an intermediate result that is a direct extension of the Cournot equilibrium. This equilibrium is used to support our new preemption equilibrium.

Lemma 2. *If the storage unit is a seller with capacity* k*, then the (symmetric) equilibrium price* p^* *and equilibrium quantities* s^* *and* q^* *under Cournot competition are:*

$$p^* = \frac{1 + \varepsilon - k}{n + 1}, \qquad s^* = k, \qquad q^* = \frac{1 + \varepsilon - k}{n + 1}. \tag{6}$$

If the storage unit is a buyer with capacity k*, then the (symmetric) equilibrium price* p^* *and equilibrium quantities* b^* *and* q^* *under Cournot competition are*

$$p^* = \frac{1 + \varepsilon + k}{n + 1}, \qquad b^* = k, \qquad q^* = \frac{1 + \varepsilon + k}{n + 1}. \tag{7}$$

The proof is trivial and therefore omitted. If the storage unit neither buys nor sells, the standard Cournot competition between n generators prevails, and then $p^* = q^* = (1+\varepsilon)/(n+1)$

in symmetric equilibrium.

For most of the paper we restrict the distribution F of shocks to be a simple, independent binary process with

$$\varepsilon \in \{a, -a\}, \; a > 0 \quad \text{and} \quad Pr(\varepsilon = a) = Pr(\varepsilon = -a) = \frac{1}{2}.$$

However we also argue that our results hold in the neighborhood of $1/2$ for an asymmetric distribution $(p, 1-p)$, and for more general Markov processes (i.e. with serial correlation) in the neighborhood of our simple distribution.

3.1 Trading energy over the long horizon

As we know from the literature on repeated games and on stochastic games (see, for example, Chatterjee et al. (2003)), these games admit many equilibria. In a companion paper Balakin and Roger (2025a), we study a more general version of the model of Section 2 and make progress by reducing the space of admissible strategies to simple heuristics and by imposing that conventional sellers play Cournot quantities every period.[6] This equilibrium is useful here to support our new preemption equilibrium.[7] What was a restriction in Balakin and Roger (2025a) becomes a standard Nash reversion play here.

Under the simple independent shock structure $(1/2, 1/2)$, and given that storage starts empty, the recursive equation may be written in the following form:

$$\begin{cases} V(0) &= \dfrac{1}{2-\beta}\left(-\dfrac{1-a+k}{n+1}\cdot k + \beta V(k)\right), \\ V(k) &= \dfrac{1}{2-\beta}\left(\dfrac{1+a-\delta k}{n+1}\cdot \delta k + \beta V(0)\right). \end{cases} \tag{8}$$

Given the nature of the stochastic process, it is immediate that V is time invariant.

Observe that it cannot be optimal for the storage operator to buy when the shock ε is

[6] The Cournot equilibrium is simple to describe, unlike any of the more sophisticated equilibrium strategies one can construct. Our last justification is the work of Bonatti et al. (2017), who study a dynamic Cournot model under incomplete information with learning. The equilibrium converges to the repeated static Nash equilibrium.

[7] The heuristic is already simplified even further since $s, b \in \{0, k\}$.

positive, nor can it be optimal to sell when it is negative. Denote the purchasing costs as B and likewise by A the revenue storage earns when selling:

$$B = B(k) = \frac{1-a+k}{n+1} \cdot k, \qquad A = A(k) = \frac{1+a-\delta k}{n+1} \cdot \delta k.$$

In both A and B the first term is the clearing price and the second one is the quantity traded. Let also the coefficients be

$$G_{01} = \left(\frac{1+a}{n+1}\right)^2, \quad G_{00} = \left(\frac{1-a+k}{n+1}\right)^2, \quad G_{10} = \left(\frac{1-a}{n+1}\right)^2, \quad G_{11} = \left(\frac{1+a-\delta k}{n+1}\right)^2.$$

G_{ij} is a (non-discounted) seller payoff when the storage is either empty ($i = 0$) or full ($i = 1$) and when the demand shock is either negative ($j = 0$) or positive ($j = 1$). Suppose also the discount factor β is such that

$$B < \frac{\beta}{2-\beta} A.$$[8]

Lemma 3. *A dynamic equilibrium exists and is characterized as follows:*

- *the empty storage buys k units with the first negative shock and sells δk units with the first positive shock afterwards;*
- *in each period, the n sellers set quantities q^* according to static Cournot competition and based on the current shock and the state of storage (full or empty). Namely,*

$$\begin{aligned} &\textit{when storage is empty:} \quad q^* = \frac{1+a}{n+1} \ \textit{if} \ \varepsilon = a, \quad q^* = \frac{1-a+k}{n+1} \ \textit{if} \ \varepsilon = -a; \\ &\textit{when storage is full:} \quad q^* = \frac{1-a}{n+1} \ \textit{if} \ \varepsilon = -a, \quad q^* = \frac{1+a-\delta k}{n+1} \ \textit{if} \ \varepsilon = a. \end{aligned}$$

Aggregate consumers' expected payments C are

$$C = \frac{1}{2(n+1)^2}\left(2n(1+a^2) - ka(n-1) - k^2\right).$$

Expected payoffs U_s of the storage unit and aggregate payoffs U_g of the producers take the

[8] Note A and B are determined in terms of primitives. This is just convenient notation.

following form:

$$U_s = \frac{1}{2}\left[-B + \frac{\beta}{2(1-\beta)}(A-B)\right], \tag{9}$$
$$U_g = \frac{1}{2}(G_{01}+G_{00}) + \frac{\beta}{4(1-\beta)}(G_{10}+G_{11}+G_{00}+G_{01}).$$

We display this equilibrium i because it is used off-path to support our cooperative equilibrium and (ii) to explore some of the salient features of the trading problem. First, storage activity increases the output of producers when the demand shock is negative. This also increases prices (when they are otherwise low); that is, storage activity is a complement to the producers. But storage also decreases the output of these producers when the shock is positive; this concurrently depresses otherwise high prices. Here storage is a substitute to producers. Furthermore, every time it engages in arbitrage, the storage unit decreases the very spread it feeds on. It is exactly this phenomenon of spread contraction that renders the exclusion equilibrium tenable and that producers exploit.

3.2 A preemption equilibrium

The equilibrium constructed in Lemma 3 gives the storage operator some leeway: while the n producers play the Cournot best response, storage moves first. Perhaps more importantly, in selecting this equilibrium (for reasons laid out in Section 3.1), it is implicitly imposed that producers accommodate the storage unit. Now we show this need not be true: there also exists an equilibrium in which the storage unit never trades–irrespective of any entry decision.

3.2.1 The equilibrium

In this cooperative equilibrium, the producers act in such a way that storage operator never finds it profitable to incur the cost purchasing in the first place. Let $\underline{\beta} := \underline{\beta}(a, n)$ and

$\overline{\beta} := \overline{\beta}(a, n, \delta, k)$ be

$$\underline{\beta} = \frac{(1+a)^2(n+1)^2}{4n(1+a^2)+(1+a)^2(n+1)^2} \quad \text{and} \quad \overline{\beta} = \frac{2}{1+\dfrac{2\delta(1+a-\delta k)}{(n+1)(1-a+2k)}}$$

Proposition 4. *Assume that*

$$\underline{\beta} \leqslant \beta \leqslant \min\left\{\overline{\beta}, 1\right\}. \tag{10}$$

Then there exists a dynamic Subgame Perfect Nash Equilibrium, such that:

- *in each period, the generators set quantities*
  - $\varepsilon = -a$*:*
    - $q^* = \frac{1-a}{2n}$ *if none of the generators deviated in the previous rounds,*
    - $q^* = \frac{1-a+k}{n+1}$ *if storage is empty and any of the generators deviated in the previous rounds,*
    - $q^* = \frac{1-a}{n+1}$ *if storage is full and any of the generators deviated in the previous rounds;*
  - $\varepsilon = a$*:*
    - $q^* = \frac{1+a}{2n}$ *if storage is empty and none of the generators deviated in the previous rounds,*
    - $q^* = \frac{1+a}{n+1}$ *if storage is empty and any of the generators deviated in the previous rounds,*
    - $q^* = \frac{1+a-\delta k}{n+1}$ *if storage is full;*
- $b = s = 0$*: storage does not trade if none of the generators have deviated; otherwise, storage trades positive quantities if*

$$B < \frac{\beta}{2-\beta}A. \tag{11}$$

The aggregate expected payments from consumers C^0 *each period are:*

$$C^0 = \frac{1+a^2}{4}.$$

The expected payoffs U_g^0 *and* U_s^0 *of the producers and the storage unit, respectively, take the following form:*

$$U_g^0 = \frac{1+a^2}{4(1-\beta)n}, \qquad U_s^0 = 0.$$

Immediately one sees that consumers and storage are worse off in this equilibrium, and the producers are better off, than in the benchmark of Lemma 3. There can be no Pareto-ranking of these equilibria, which speaks to the conflict that is inherent to this game.

In intuitive, economic terms, storage is preempted because of the combination of two factors: *(i)* it starts empty and so must first buy to become active, and *(ii)* its marginal cost is determined by the other players.[9]

More precisely, in this equilibrium, producers collude to the joint-profit maximizing quantities and the storage unit never buys. Equilibrium play must deter two kinds of deviations. First, the producers must elect to not deviate; this is supported by the threat of Cournot reversion, which is subgame perfect as established by Lemmata 2 and 3. This is exactly what the left-hand side of Condition (10) delivers: the discount factor must be large enough, as is well known. The second kind of deviation is novel: as long as no producer deviates, the storage operator also prefers not purchasing. If she does buy at the monopoly price, the sellers *(i)* play the quantities described in Lemma 3 when storage is in a position to sell, and *(ii)* revert to the joint monopoly quantity when storage must buy again. This is also subgame perfect. This threat is sufficient because the purchasing cost is too high compared to what the storage unit can collect once sellers respond; this is the right-hand side of Condition (10), which is an upper bound on the discount factor. This condition only needs considering payoffs for one trading cycle: as long as no producer deviates, they all revert to the joint-monopoly

[9]Of course, so is its marginal revenue, but this is true in any standard strategic situation.

quantities; that is, players are back in the original situation. Note also that here the strategic effect works through market power to support the equilibrium. Absent market power, this equilibrium does not exist.

This equilibrium is more intricate than the simple grim trigger strategy. Nash reversion is part of the equilibrium (to deter deviations by sellers), but that is not enough. To deter the storage unit from buying, off-path play must cycle between two quantities depending on the state of the system. That state is summarized by the pair (c, ε) of the state of charge and the demand shock.

To be sure, this equilibrium is also not in the spirit of a folk theorem in that the discount factor β must be bounded above–and that upper bound $\overline{\beta}(n, a, \delta, k)$ is not trivially 1. This function $\overline{\beta}$ decreases in a and δ and increases in k and n, both of which being intuitive. Its lowest value is 0 when $a \to 1$ and $k \to 0$ (no market power), but it changes drastically for small changes in parameters: for example, for $k = 0,2$ and $a = 0.8$, the lowest value of $\overline{\beta}$ already jumps to 0.72 with $\delta = 1, n = 2$. For low $a = 0.2$ and $n = 2, \delta = 0.95$ our equilibrium exists for $0.61 \leqslant \beta \leqslant 1$ under any k; for higher $a = 0.6$ and $n = 2, \delta = 0.95, k = 0.1$, one needs $0.68 \leqslant \beta \leqslant 0.77$ but for $a = 0.6, n = 2, \delta = 0.95$ and any $k > 0.24$, the upper bound $\overline{\beta}$ hits its natural boundary again: $0.68 \leqslant \beta \leqslant 1$. The left-hand side of (10) is simpler to understand: it is a function $\underline{\beta}(a, n)$ that increases in both its arguments. This function is bounded below by 9/17 when $n = 2$ and $a \to 0$ and reaches 1 for $n \to \infty$; for $a = 1, n = 2$, we have $\underline{\beta} = 9/13$. That is, the condition $\underline{\beta}(n, a) \leqslant \beta$ is somewhat demanding: it exceeds the standard Bertrand condition with unit demand.

Proposition 4 suggests the emergence of storage is not a foregone conclusion, even absent entry costs. Facing such a situation, and in light of the equilibrium payoffs of Proposition 4, policy makers may be compelled to intervene, especially if guided by a consumer welfare standard. Preemption may be overcome with the help of a small subsidy, however in perpetuity. Here it is enough to cover the revenue shortfall to induce storage to operate; but since a single cycle is sufficient to preempt storage, that subsidy may have to be paid out every period. In contrast, a lump-sum payment equal to the prevent value of losses in perpetuity fails to provide any incentive for storage to operate. This is not very different from the perennial subsidization of public transport. Returning to our motivating energy market, this form of preemption continues to be relevant even as thermal generators exit the market: it can be carried out by incumbent storage units instead.

3.2.2 Robustness

In our construction of Proposition 4 we limit the stochastic process to be symmetric and serially independent. A natural question to ask is whether this is limiting; the answer is no. Of course, the details of Condition (10) differ but the construction remains. One can depart from the simple symmetric, i.i.d. case in two ways.

First, one can enrich the Markovian structure, as we do in our companion paper Balakin and Roger (2025a). Then Condition (11) changes to

$$B < \frac{\beta(1-y)}{1-\beta y}A,$$

where $y = Pr\{\varepsilon_{t+1} = -a|\varepsilon_t = -a\}$ is a level of persistence (serial correlation). If $y = 1/2$ we revert to Proposition 4. Continuity of the functions $\underline{\beta}, \overline{\beta}$ guarantees the same equilibrium continues to exist in the neighborhood of $y = 1/2$. For $y > 1/2$, shocks are persistent; this is bad news for storage because it needs to wait for longer to sell (so we expect $\overline{\beta}$ to increase). For $y < 1/2$, the converse is true: shocks change faster than in the independent case, so storage expects to wait less. This renders purchasing more attractive for the storage unit. In this case, we can also expect $\overline{\beta}$ to decrease. It is a mere exercise to compute the functions $\underline{\beta}$ and $\overline{\beta}$ for this more general process and then to check the construction.

Second, one can consider asymmetric shocks $(p, 1-p)$, $p \neq 1/2$ that are serially independent. The same continuity argument applies and the equilibrium continues to exist for p in the neighborhood of 1/2. Away from 1/2, it is also a mere exercise in algebra to compute the functions $\underline{\beta}$ and $\overline{\beta}$ and to check that the equilibrium exists for some parameter values or to put restrictions on p for it to exist. If the negative shock is more likely, say $p > 1/2$ selling takes longer, so one can expect $\overline{\beta}$ to increase.

4 Conclusion

In this short paper we construct an exclusion equilibrium in which conventional producers can collude to prevent a storage unit from being active. This result requires both market power, so that players strategically determine clearing prices, and that the storage unit starts the game empty. In equilibrium, it stays empty. The equilibrium combines a simple grim-trigger strategy that disciplines the sellers with a threat of cycling between a high purchase price and a low selling price.

We believe this is a plausible equilibrium in that (i) storage activity is a stronger substitute than a complement for producers because it sells in periods of high demand, so the producers have incentives to exclude storage operators, and (ii) many industries feature market power; that is, are strategic environments.

As storage emerges as the key technology of the energy transition, policy makers should be aware of the potential pitfalls this novel class of strategic situation can induce. There is still a tremendous amount of work to do to really understand the economics of storage. This is but an example of a bad outcome and of its potential remedy.

A Appendix – for online publication

A.1 Proof of Lemma 3

According to (7) and (6), the storage operator buys k units under price $p = (1 - a + k)/(n+1)$ and sells δk units under price $p = (1 + a - \delta k)/(n+1)$. The probability that the storage unit observes the first positive shock after i periods is $(1/2)^{i-1} \cdot (1/2)$. Thus, the total discounting of waiting for the positive shock after recharging is equal to

$$\frac{1}{2} \cdot \beta + \frac{1}{4} \cdot \beta^2 + \cdots + \frac{1}{2^i} \cdot \beta^i + \cdots = \frac{\beta}{2-\beta}.$$

Hence trading is profitable for the storage operator if $-B + \frac{\beta}{2-\beta} A > 0$.

Four possible deviations of the storage unit should be considered. All other deviations are just compositions of those four.

- The unit is full but deviates by not selling under the positive shock. Then there may be only loss comparing to the default strategy. Indeed, nothing changes on the market except the future profits to be discounted by β.

- The unit is empty and deviates by not buying under the negative shock. Also, no gains here.

- The unit is full and deviates by selling under the negative shock. In this situation, the quantities supplied by the producers are $q = (1 - a)/(n + 1)$. The resulting price after the deviation is

  $$p = 1 - a - \delta k - n\frac{1-a}{n+1} = \frac{1-a}{n+1} - \delta k.$$

  To make this deviation profitable, the storage operator must gain more than if it waits for the positive shock and sells in that period:

  $$\left(\frac{1-a}{n+1} - \delta k\right) \delta k > \frac{\beta}{2-\beta} \frac{1 + a - \delta k}{n+1} \delta k,$$

  which is impossible when $B < \frac{\beta}{2-\beta} A$.

- The unit is empty and deviates by buying under the positive shock. Here we have $q = (1+a)/(n+1)$, and the resulting price after the deviation is $p = (1+a)/(n+1)+k$. The profits after selling the purchased energy are:

$$
\begin{aligned}
& -\left(\frac{1+a}{n+1}+k\right)k + \frac{\beta}{2-\beta}\frac{1+a-\delta k}{n+1}\delta k \\
= & -\frac{k}{n+1}\left(\left(1-\frac{\beta}{2-\beta}\delta\right)(1+a) + \left(\frac{\beta}{2-\beta}\delta^2+n+1\right)k\right) < 0.
\end{aligned}
$$

  There are no gains from this deviation.

Ruling out deviations of the producers is simple. In each round, we have a static Cournot equilibrium for all the participants. Thus, any change of the equilibrium quantity in any round t leads to a decrease in the payoffs in that round and thus to a decrease in the overall payoffs. Indeed, the stage-game Cournot equilibrium is also an equilibrium in the long-horizon game.

To find the expected payoffs of the storage unit, let us introduce value function $V_t(i),\ i \in \{1,0\}$. $V_t(i)$ is the total expected payoffs of the unit from moment t if the current state is full $(i=1)$ or empty $(i=0)$. We have a system of recursive equations:

$$
\begin{cases}
V_t(1) = \frac{1}{2}\cdot(A+\beta V_{t+1}(0)) + \frac{1}{2}\cdot\beta\cdot V_{t+1}(1), \\
V_t(0) = \frac{1}{2}\cdot\beta\cdot V_{t+1}(0) + \frac{1}{2}\cdot(-B+\beta\cdot V_{t+1}(1)).
\end{cases}
$$

It can be rewritten in matrix form

$$
V_t = P + \beta\cdot Q\cdot V_{t+1}, \tag{12}
$$

where

$$
V_t = \begin{pmatrix} V_t(1) \\ V_t(0) \end{pmatrix}, \qquad P = \frac{1}{2}\begin{pmatrix} A \\ -B \end{pmatrix}, \qquad Q = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.
$$

Note that $Q^2 = Q$. For $\beta < 1$, we can find from (12) that

$$V_0 = P + \sum_{i=1}^{t} \beta^i Q^i \cdot P + \beta^{t+1} Q^{t+1} \cdot V_{t+1} =$$

$$= P + \frac{\beta(1-\beta^t)}{1-\beta} \cdot Q \cdot P + \beta^{t+1} \cdot Q \cdot V_{t+1} \xrightarrow[t\to\infty]{} P + \frac{\beta}{1-\beta} \cdot Q \cdot P = \frac{1}{4} \begin{pmatrix} \frac{2-\beta}{1-\beta} A - \frac{\beta}{1-\beta} B \\ \frac{\beta}{1-\beta} A - \frac{2-\beta}{1-\beta} B \end{pmatrix}. \tag{13}$$

The lower term is exactly U_s.

To find the expected payoffs of the producers, let us introduce value function $W_t(i)$, $i \in \{1, 0\}$. $W_t(i)$ is the total expected payoffs of a generator from moment t if the current state of the storage unit is full ($i = 1$) or empty ($i = 0$). We have a system of recursive equations:

$$\begin{cases} W_t(1) = \frac{1}{2} \cdot (G_{10} + \beta W_{t+1}(1)) + \frac{1}{2} \cdot (G_{11} + \beta \cdot W_{t+1}(0)), \\ W_t(0) = \frac{1}{2} \cdot (G_{00} + \beta \cdot W_{t+1}(1)) + \frac{1}{2} \cdot (G_{01} + \beta \cdot W_{t+1}(0)) \end{cases}$$

that can be rewritten in matrix form

$$W_t = R + \beta \cdot Q \cdot W_{t+1}, \quad \text{where}$$

$$W_t = \begin{pmatrix} W_t(1) \\ W_t(0) \end{pmatrix}, \qquad R = \frac{1}{2} \begin{pmatrix} G_{10} + G_{11} \\ G_{00} + G_{01} \end{pmatrix}, \qquad Q = Q^2 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Using the same algebra as for $V(t)$ earlier, we obtain

$$W_0 \xrightarrow[t\to\infty]{} R + \frac{\beta}{1-\beta} \cdot Q \cdot R = \begin{pmatrix} \frac{1}{2}(G_{10} + G_{11}) + \frac{\beta}{4(1-\beta)}(G_{10} + G_{11} + G_{00} + G_{01}) \\ \frac{1}{2}(G_{00} + G_{01}) + \frac{\beta}{4(1-\beta)}(G_{10} + G_{11} + G_{00} + G_{01}) \end{pmatrix}.$$

The lower term is exactly U_g.

To find aggregate consumers' expected payments C, let us introduce the value function

$Z_t(i)$, $i \in \{1,0\}$. $Z_t(i)$ is the aggregate expected payments of consumers from moment t if the current state of the storage unit is full ($i=1$) or empty ($i=0$). We have a system of recursive equations:

$$\begin{cases} Z_t(1) = \frac{1}{2} \cdot (C_1 + \beta Z_{t+1}(1)) + \frac{1}{2} \cdot (C_2 + \beta \cdot Z_{t+1}(0)), \\ Z_t(0) = \frac{1}{2} \cdot (C_3 + \beta \cdot Z_{t+1}(1)) + \frac{1}{2} \cdot (C_4 + \beta \cdot Z_{t+1}(0)), \end{cases}$$

where

$$C_1 = \frac{1-a}{n+1} \cdot \frac{1-a}{n+1} n, \qquad C_2 = \frac{1+a-\delta k}{n+1} \cdot \left(\frac{1+a-\delta k}{n+1} n + \delta k \right),$$
$$C_4 = \frac{1+a}{n+1} \cdot \frac{1+a}{n+1} n, \qquad C_3 = \frac{1-a+k}{n+1} \cdot \left(\frac{1-a+k}{n+1} n - k \right).$$

It can be rewritten in matrix form

$$Z_t = \overline{C} + \beta \cdot Q \cdot Z_{t+1}, \quad \text{where}$$

$$Z_t = \begin{pmatrix} Z_t(1) \\ Z_t(0) \end{pmatrix}, \qquad \overline{C} = \frac{1}{2} \begin{pmatrix} C_1 + C_2 \\ C_3 + C_4 \end{pmatrix}, \qquad Q = Q^2 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Using the same algebra as for V_t and W_t earlier, we obtain

$$Z_0 \xrightarrow[t \to \infty]{} \overline{C} + \frac{\beta}{1-\beta} \cdot Q \cdot \overline{C} = \frac{1}{4(1-\beta)} \begin{pmatrix} (2-\beta)(C_1 + C_2) + \beta(C_3 + C_4) \\ \beta(C_1 + C_2) + (2-\beta)(C_3 + C_4) \end{pmatrix}.$$

The lower term after substituting C_i is exactly C.

A.2 Proof of Proposition 4

Let us start with the possible deviation of the storage operator. If the storage unit decides to operate and make a purchase under monopolistic quantities set by the producers, the equilibrium prescribes that it sells under Cournot quantities. The expected storage payoffs during one round-trip cycle are

$$-\left(\frac{1-a}{2}+k\right)k+\frac{\beta}{2-\beta}\frac{1+a-\delta k}{n+1}\delta k, \tag{14}$$

and so are non-positive if

$$\frac{\beta}{2-\beta}\leqslant\frac{(n+1)\left(\frac{1-a}{2}+k\right)}{\delta\left(1+a-\delta k\right)},$$

which is equivalent to the right side of (10). Hence, as long as producers can maintain their collusive equilibrium, the storage unit should not purchase anything and so never operates. Condition (14) is sufficient as long as no seller deviates because the equilibrium prescribes they revert to the joint-monopoly quantity when $\varepsilon=-a$.

Next we turn to the possible deviations of a producer. First, we consider the case where the inequality (11) does not hold; then storage finds it unprofitable to operate with Cournot bids. This is the condition of Lemma 3. A producer deviates from its monopolistic quantity when the shock is positive: $\varepsilon=a$. Let the deviation be γ. Then the payoffs of the producer after deviating is

$$\left(\frac{1+a}{2n}+\gamma\right)\left(\frac{1+a}{2}-\gamma\right)+\frac{\beta}{1-\beta}\left(\frac{1}{2}\left(\frac{1+a}{n+1}\right)^2+\frac{1}{2}\left(\frac{1-a}{n+1}\right)^2\right).$$

The producer increases its quantity by γ, which also results in a price decrease by γ. It also triggers the punishment regime: all other sellers switch from monopolistic quantities to Cournot quantities, which results in the discounted expected payoff over infinite horizon expressed by the second item. This deviation is unprofitable if

$$\begin{aligned}&\left(\frac{1+a}{2n}+\gamma\right)\left(\frac{1+a}{2}-\gamma\right)+\frac{\beta}{1-\beta}\left(\frac{1}{2}\left(\frac{1+a}{n+1}\right)^2+\frac{1}{2}\left(\frac{1-a}{n+1}\right)^2\right)\\ \leqslant\ &\frac{1+a}{2n}\frac{1+a}{2}+\frac{\beta}{1-\beta}\left(\frac{1}{2}\frac{(1+a)^2}{4n}+\frac{1}{2}\frac{(1-a)^2}{4n}\right),\end{aligned}$$

which can be simplified to

$$\frac{n-1}{n}(1+a)\gamma - 2\gamma^2 \leqslant \frac{\beta}{1-\beta}\frac{(1+a^2)(n-1)^2}{2n(n+1)^2}.$$

The maximum on the left side can be achieved when $\gamma_{max} = (1+a)(n-1)/(4n)$. Then we obtain

$$\frac{(1+a)^2}{4n} \leqslant \frac{\beta}{1-\beta}\frac{(1+a^2)}{(n+1)^2}, \tag{15}$$

which is equivalent to the left side of (10).

Now suppose that a producer deviates from the joint-monopoly quantity when the shock is negative: $\varepsilon = -a$. This move is unprofitable if

$$\begin{aligned} &\left(\frac{1-a}{2n}+\gamma\right)\left(\frac{1-a}{2}-\gamma\right)+\frac{\beta}{1-\beta}\left(\frac{1}{2}\left(\frac{1+a}{n+1}\right)^2+\frac{1}{2}\left(\frac{1-a}{n+1}\right)^2\right) \\ \leqslant\ &\frac{1-a}{2n}\frac{1-a}{2}+\frac{\beta}{1-\beta}\left(\frac{1}{2}\frac{(1+a)^2}{4n}+\frac{1}{2}\frac{(1-a)^2}{4n}\right), \end{aligned}$$

which can be simplified to

$$\frac{(1-a)^2}{4n} \leqslant \frac{\beta}{1-\beta}\frac{1+a^2}{(n+1)^2}.$$

This inequality is weaker than (15).

Second, consider the case where inequality (11) holds: storage can operate under Cournot quantities, as in Lemma 3; that is, here, immediately after any of the producers deviates. Since the payoffs of a deviating producer are lower when storage participates than when it does not, inequality (15) is sufficient to make this deviation unprofitable.

Finally, we should also prove that our pool of equilibrium strategies forms an SPNE even at information sets that are off the equilibrium path. Namely, producers must not want to deviate even if storage operates. Indeed, if the shock is positive, all the producers set static Cournot quantities, and it becomes unprofitable to deviate. If the shock is negative, any deviation from the monopolistic quantities implies a punishment that was already considered earlier. Hence, all the possible deviations of a producer are unprofitable, and the proposed strategies of all the players form a Nash equilibrium.

To calculate the expected payoffs of a producer, we obtain a recursive equation for the payoffs Z_t of sellers, starting from the period t:

$$Z_t = \frac{1}{2}G_{01} + \frac{1}{2}G_{10} + \beta Z_{t+1}.$$

We can easily find $Z_0 = U_g^0$ from this equation.

Since positive and negative shocks are equally likely, the aggregate consumers' expected payments per period are

$$C^0 = \frac{1}{2} \cdot \left(\frac{1+a}{2}\right)^2 + \frac{1}{2} \cdot \left(\frac{1-a}{2}\right)^2 = \frac{1+a^2}{4}.$$

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