Storage Cycles

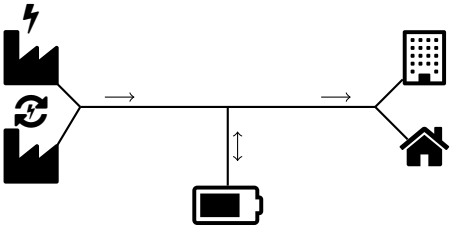
Sergei Balakin Guillaume Roger

Monash University

Introduction 1

Storage

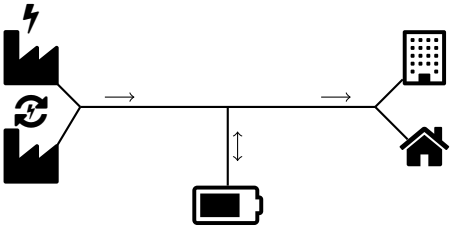
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- has a limited capacity
- can sway market power



Introduction 1

Storage

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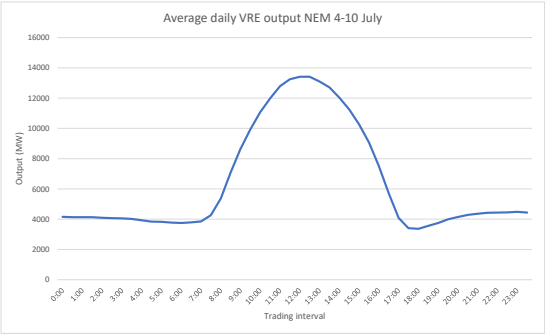


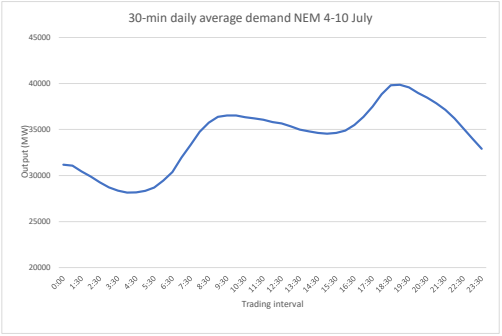
Question: what is the optimal dynamic strategy of a storage operator?

- *when* and *how intensely* does it (dis)charge (power rate)?
- how does it depend on capacity? on shocks?
- how does it depend on dynamics?

Introduction 2

Mismatch between VRE (grid-scale) supply (left) and (gross) energy demand (right).
Daily 30-min averages, 4-10 July 2024, Australia (AEMO data)



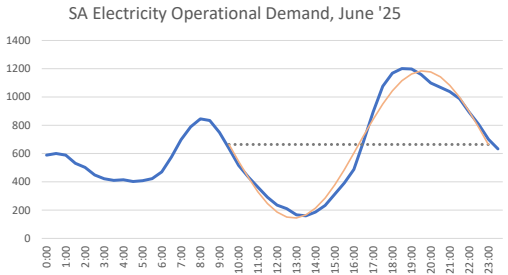


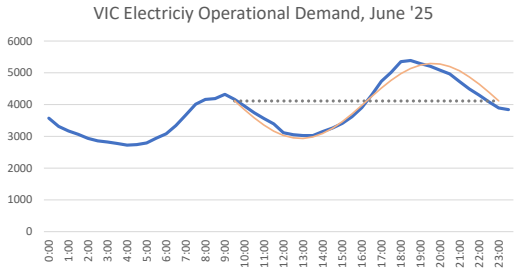
Introduction 3

Battery storage history:

- 2016: Mira Loma, CA (80 MWh); Toronto, ON (53)
- 2017: Kauai, HI (52); Hornsdale, SA (129 $\nearrow_{2020}$ 194)
- 2018: Hertfordshire, UK (50)
- 2020: Moss Landing Vistra, CA (1200 $\nearrow_{2021}$ 1600 $\nearrow_{2023}$ 3000 $\searrow_{2025}$ 1800)
- 2021: Victorian Big Battery, VIC (450); Manatee Energy Storage, FL (900)
- 2022: Valley Center, CA (560); Slate, CA (561); Moss Landing Elkhorn, CA (730); Diablo Energy Storage, CA (800); Desert Sunlight, CA (920); Lancaster Area Battery, CA (508); Crimson, CA (1400)
- 2023: Oberon, CA (1000); Kenhardt, South Africa (1140); Kapolei, HI (565); Desert Peak Energy Storage I, CA (1300)
- 2024: Gemini, NV (1416); Eleven Mile, AZ (1200); The Red Sea Project, Saudi Arabia (1300); Edwards Sanborn, CA (3287)

Introduction 4



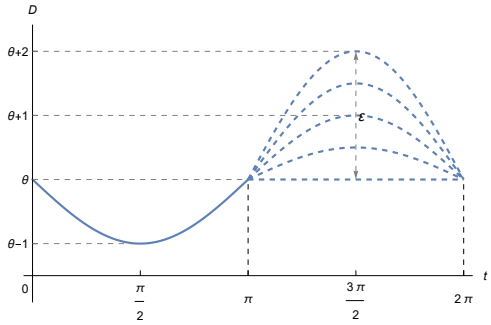


Outline

- The model
- One period
- Two periods
- No information case
- Welfare implications
- Literature

The Model: Demand

$$D(t,\varepsilon) = \begin{cases} \theta - \sin t & \text{if } \ t \in [0,\pi), \\ \theta - (1+\varepsilon)\sin t & \text{if } \ t \in [\pi, 2\pi) \end{cases}$$



- $\theta \geqslant 1$ is a parameter, t is time, and $\varepsilon \sim \mathsf{U}_{[-1,1]}$ is a demand shock
- D is periodic: $\mathbb{E}_{\varepsilon} D(t,\varepsilon) = \mathbb{E}_{\varepsilon_i} D(t + 2\pi i, \varepsilon_i)$, shocks are iid

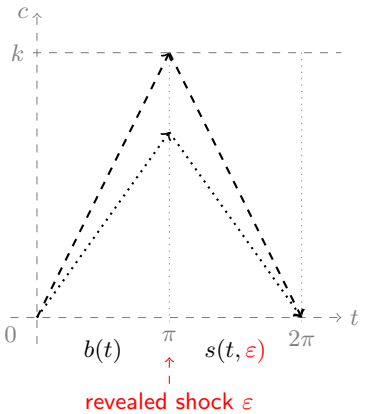
The Model: Storage & Generators

- STORAGE:
  - Storage unit has limited capacity k
  - Storage operator buys flows $b(t)$ under $t \in [0, \pi)$ and sells $s(t)$ under $t \in [\pi, 2\pi)$
- GENERATORS:
  - Each seller produces quantity q with costs $c(q) := q^2/2$
  - Perfect competition:

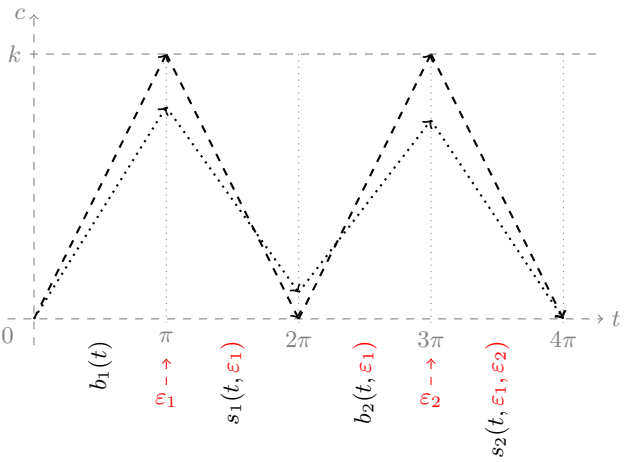
$$\begin{aligned} q(t) &= p(t) = \theta - \sin t + b(t), && t \in [0, \pi), \\ q(t, \varepsilon) &= p(t, \varepsilon) = \theta - (1 + \varepsilon) \sin t - s(t), && t \in [\pi, 2\pi). \end{aligned}$$

Model: Timing

One period:



Two periods (cycles):



One cycle

$$\max_{b(t),\{s(t,\varepsilon)\}} \left[-\int_0^{\pi} (\theta - \sin t + b(t))\, b(t) dt + \frac{1}{2} \int_{-1}^{1} \int_{\pi}^{2\pi} (\theta - (1+\varepsilon)\sin t - s(t,\varepsilon))\, s(t,\varepsilon) dt d\varepsilon \right] \quad (1)$$

subject to

$$\begin{aligned} & b(t) \geqslant 0, \qquad s(t,\varepsilon) \geqslant 0, \\ & \int_0^{\pi} b(t) dt \leqslant k, \qquad \int_0^{2\pi} (b(t) - s(t,\varepsilon))\, dt \geqslant 0 \qquad \text{for any } \varepsilon \in [-1,1]. \end{aligned} \quad (2)$$

One cycle, $k \leqslant 1$

Lemma (One cycle)

The optimal storage strategy that solves (1) *subject to* (2) *is characterized as:*

- *If* $k \leqslant 1$,

$$b(t) = \begin{cases} \frac{1}{2}(\sin t - \sin t_0) & \text{if } t \in [t_0, \pi - t_0), \\ 0 & \text{otherwise,} \end{cases}$$

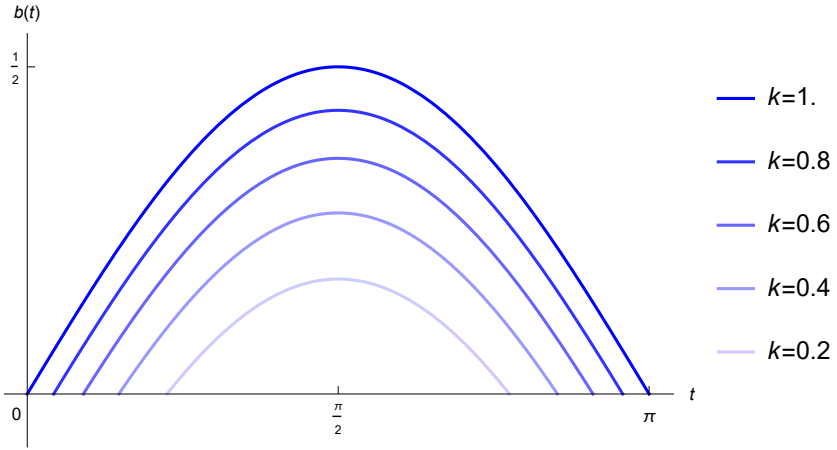
$$s(t,\varepsilon) = \begin{cases} -\frac{1+\varepsilon}{2}(\sin t + \sin t_\varepsilon) & \text{if } t \in [\pi + t_\varepsilon, 2\pi - t_\varepsilon), \\ 0 & \text{otherwise,} \end{cases} \qquad \text{if} \quad \varepsilon \geqslant k-1,$$

$$s(t,\varepsilon) = \begin{cases} \frac{1}{\pi}(k-1-\varepsilon) - \frac{1}{2}(1+\varepsilon)\sin t & \text{if } t \in [\pi, 2\pi), \\ 0 & \text{otherwise,} \end{cases} \qquad \text{if} \quad \varepsilon < k-1,$$

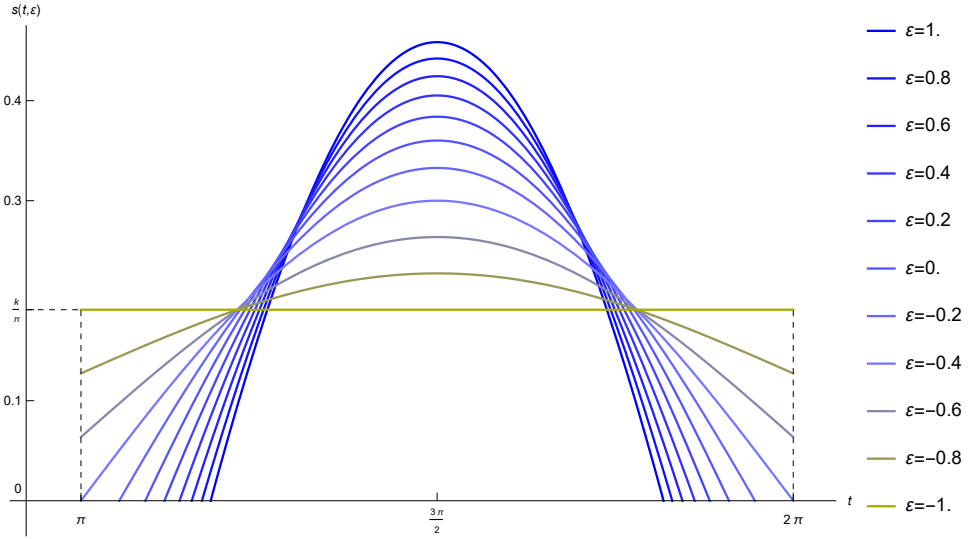
where $t_0, t_\varepsilon \in [0, \pi/2]$ *are defined by*

$$\cos t_0 - \left(\frac{\pi}{2} - t_0\right)\sin t_0 = k, \qquad \cos t_\varepsilon - \left(\frac{\pi}{2} - t_\varepsilon\right)\sin t_\varepsilon = \frac{k}{1+\varepsilon}.$$

One cycle, optimal $b(t)$ for different k



One cycle, optimal $s(t, \varepsilon)$ for $k = 0.6$ and different ε



One cycle, $1 < k \leqslant c_m$

Lemma (One cycle)

- *If* $1 < k \leqslant c_m \approx 1.009$,

$$b(t) = \begin{cases} \frac{\sin t}{2} + \frac{1}{\pi}(k-1) & \text{if } t \in [0, \pi], \\ 0 & \text{otherwise,} \end{cases}$$

$$s(t, \varepsilon) = \begin{cases} -\frac{1+\varepsilon}{2}(\sin t + \sin t_\varepsilon) & \text{if } t \in [\pi + t_\varepsilon, 2\pi - t_\varepsilon), \\ 0 & \text{otherwise,} \end{cases} \qquad \text{if} \quad \varepsilon \geqslant k-1,$$

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One cycle, $k > c_m$

Lemma (One cycle)

- *If* $k > c_m$,

$$b(t) = \begin{cases} \frac{\sin t}{2} + \frac{1}{\pi}(c_m - 1) & \text{if } t \in [0, \pi], \\ 0 & \text{otherwise,} \end{cases}$$

$$s(t, \varepsilon) = \begin{cases} -\frac{1+\varepsilon}{2}\left(\sin t + \sin \bar{t}_\varepsilon\right) & \text{if } t \in [\pi + \bar{t}_\varepsilon, 2\pi - \bar{t}_\varepsilon), \\ 0 & \text{otherwise,} \end{cases} \qquad \text{if} \quad \varepsilon \geqslant c_m - 1,$$

$$s(t, \varepsilon) = \begin{cases} \frac{1}{\pi}\left(c_m - 1 - \varepsilon\right) - \frac{1}{2}\left(1 + \varepsilon\right)\sin t & \text{if } t \in [\pi, 2\pi), \\ 0 & \text{otherwise,} \end{cases} \qquad \text{if} \quad \varepsilon < c_m - 1,$$

where $\bar{t}_\varepsilon \in [0, \pi/2]$ *is defined by*

$$\cos t - \left(\frac{\pi}{2} - t\right)\sin t = \frac{c_m}{1 + \varepsilon}.$$

Two cycles

$$\max_{b_1,\{s_1\},\{b_2\},\{s_2\}} \left[-\int_0^{\pi} (\theta - \sin t + b_1(t))\, b_1(t) dt + \frac{1}{2} \int_{-1}^{1} \left(\int_{\pi}^{2\pi} (\theta - (1+\varepsilon_1)\sin t - s_1(t,\varepsilon_1))\, s_1(t,\varepsilon_1) dt \right.\right.$$

$$\left.\left. - \int_{2\pi}^{3\pi} (\theta - \sin t + b_2(t,\varepsilon_1))\, b_2(t,\varepsilon_1) dt + \frac{1}{2} \int_{-1}^{1} \int_{3\pi}^{4\pi} (\theta - (1+\varepsilon_2)\sin t - s_2(t,\varepsilon_1,\varepsilon_2))\, s_2(t,\varepsilon_1,\varepsilon_2) dt d\varepsilon_2 \right) d\varepsilon_1 \right]$$

subject to

$$b_1(t) \geqslant 0, \quad s_1(t,\varepsilon_1) \geqslant 0, \quad b_2(t,\varepsilon_1) \geqslant 0, \quad s_2(t,\varepsilon_1,\varepsilon_2) \geqslant 0, \qquad \int_0^{\pi} b_1(t) dt \leqslant k,$$

$$\int_0^{2\pi} (b_1(t) - s_1(t,\varepsilon_1))\, dt \geqslant 0, \qquad \int_0^{3\pi} (b_1(t) - s_1(t,\varepsilon_1) + b_2(t,\varepsilon_1))\, dt \leqslant k,$$

$$\int_0^{4\pi} (b_1(t) - s_1(t,\varepsilon_1) + b_2(t,\varepsilon_1) - s_2(t,\varepsilon_1,\varepsilon_2))\, dt \geqslant 0 \qquad \text{for any } \varepsilon_1, \varepsilon_2 \in [-1,1].$$

Two cycles, $k \leqslant c_m$

Proposition (Two cycles)

First round:

- *buy* $\min\{k, \bar{c}_m\}$, *where* $\bar{c}_m \geqslant c_m$
- *sell according to the figure* $\hookrightarrow$

We are left with $c \geqslant 0$.

Second round:

- *buy* $\min\{k, \underline{c}\}$, *where* $\underline{c} := \underline{c}(c)$: $c_m - c \leqslant \underline{c}(c) \leqslant c_m$
- *sell everything*

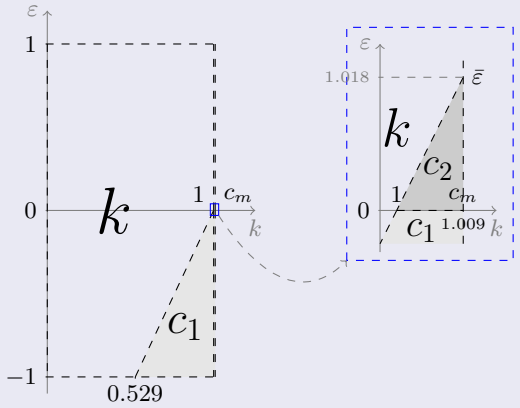
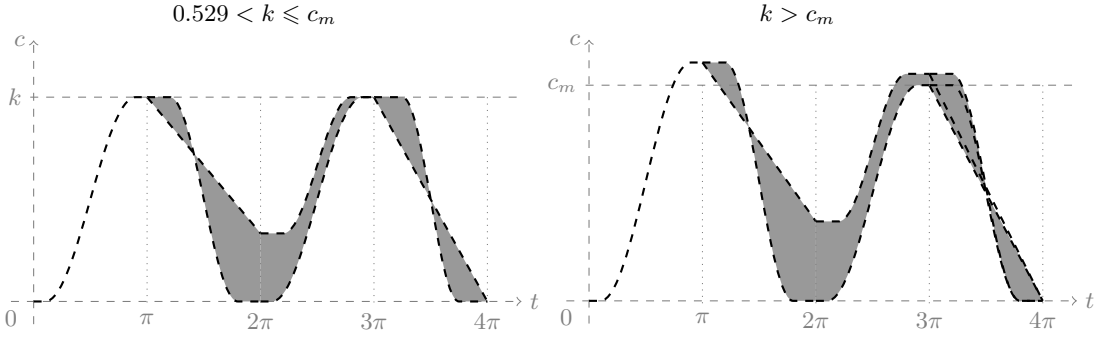
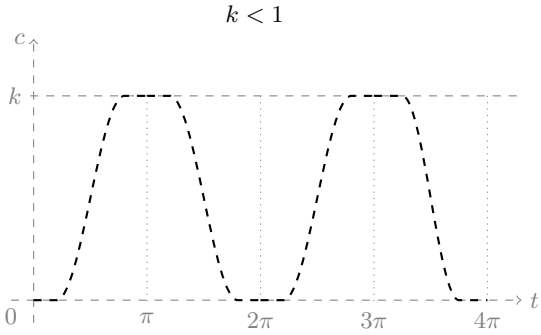


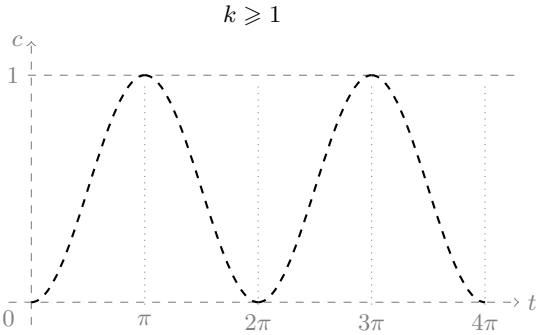
Figure: Selling scheme in the first period

Two cycles, dynamics



Benchmark: no information (ε is not revealed)





Welfare Implications (one period)

- W_0, W_1 - consumers' payments in the case of without/with information

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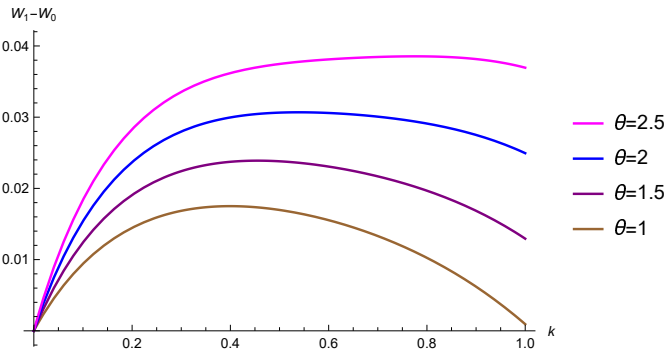
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- $W_0(k), W_1(k) \searrow$: storage smooths prices and increases consumer welfare
- $W_1 > W_0$: consumers are always worse off when storage holds better information
- $W_1 - W_0$ is not monotone!



Literature

- Storage with *market power*:
  - Karaduman (2023)
  - Williams & Green (2022)
  - Geske & Green (2020)
  - Balakin & Roger (2025a) & (2025b)
- Storage with *power rates*
  - Andres-Cerezo & Fabra (2023a)
- Power rates vs. quantities
  - Balakin & Roger (2026)
- Asset trading
  - Vayanos (1999)
  - Almgren & Criss (2001)
  - Sannikov & Skrzypacz (2023)

Conclusion

- We study the profit-maximizing charge and discharge strategy of a monopoly storage operator in a competitive wholesale electricity market
  - in continuous time;
  - with uncertain, inelastic, periodic demand;
  - under different schemes of information extraction.
- The storage unit adjusts both the time interval and the rate of operation (*when* and *how*) to maximize its profits.
- Also, when the capacity is sufficiently large (and the shock is small), intertemporal linkages between cycles emerge (saving energy for future).
- Welfare implications: better-informed storage extracts more surplus from consumers, but this difference is not monotone with k